



SubBytes Circuit Design Submission

(Level 1)

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1. SubBytes

1.1. Introduction

The SubBytes is one of the transformation repeatedly used in AES algorithm. The Rijndael algorithm, announced as Advanced Encryption Standards (AES), encrypts information through loops of ShiftRows, SubBytes, MixColumns, and AddRoundKey. Besides from this loops (called round), the key scheduled for each AddRoundKey is generated by also applying SubBytes several times during the key expansion routine [3].

SubBytes circuit (S-Box) costs outmost in circuits performing AES. Reducing the delay of the S-Box, which is the slowest function block of the entire circuit, as well as reducing its size are then encouraged.

1.2. Architecture Overview

1.2.1. Top Level Circuit Block

The submitted SubBytes transform circuit (S-Box) converts 8 bits data (XIN) to other 8 bits (YOUT). In S-Box mode, the circuit performs a multiplicative inversion over Galois Field $GF(2^8)$ followed by an affine transformation. The inverse transformation is performed in S-Box⁻¹ mode of the circuit: inverse affine followed by the same $GF(2^8)$ inversion. A control signal (INV) is then used to switch between these modes.

Table 1-1. Top view pin lists.

SUBBYTES			
signal name	In/Out	Bit width	Explanation
CLK	IN	1	clock input
RESET	IN	1	assertion '1' means reset
XIN	IN	8	input data
INV	IN	1	'0' means S-Box mode, '1' means S-Box ⁻¹ mode
YOUT	OUT	8	transformed output

Circuit sharing between S-Box and S-Box⁻¹ is used provided that both operations need the multiplicative inversion $GF(2^8)$. The circuit block diagram is shown in Figure 1-1. The data path is registered in the $GF(2^8)$ inverter output. Thus, here, we have synchronous inverse $GF(2^8)$ output sampled at CLK rising edge for the next path. During reset this section feeds the next path with "0000 0000". Signals RESET and INV are of asynchronous type.

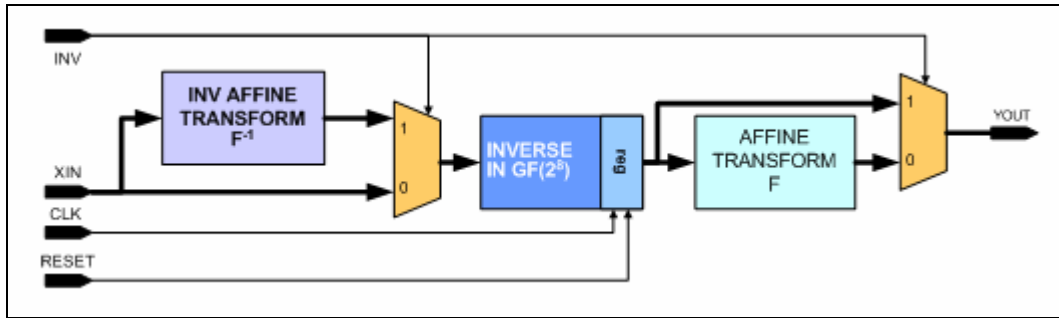


Figure 1-1. Circuit block diagram.

1.2.2. Inverse in $GF(2^8)$

Inverse $GF(2^8)$ circuit dominates the speed/area cost of the whole S-Box. Various methods for constructing compact inversion circuit were developed e.g. Itoh Tsujii, direct S-Box from look up-table, SOP, POS, BDD, and the composite field technique. The later method effectively reduces these costs through a compact multiplicative inversion circuit. It computes inversion over a field A ($GF(2^8)$), where A, with the irreducible polynomial $m(x) = x^8 + x^4 + x^3 + x + 1$, is extended from $GF(2)$ (Figure 1-2). The method is sequenced in the following steps [4]:

- (1) Mapping of all field A elements to composite field B using an isomorphism function δ .
- (2) Computation of the multiplicative inverses over the field B.
- (3) Re-mapping of the computation results to A using δ^{-1} .

The term “mapping” is simply reflected through *slicing* of the 8 bits input vector. Slicing allows for choice of composite fields, cheaper (less than 8-bit) operation in smaller field, parallel processing of the slices, conversion between representation in GF , and data rearrangement [1]. Hence, a single degree extension of $GF(2^8)$ will slice the input bits into 4 bits-based computations.

The performance of the data-slicing depends on choosing the right “slice”. Moreover, determination of the irreducible polynomials used (from all possible choices) is done by calculating the complexities e.g.: space complexity (area) is measured by the number of logical XOR and AND. The following irreducibles and constants were chosen in [4] as optimum.

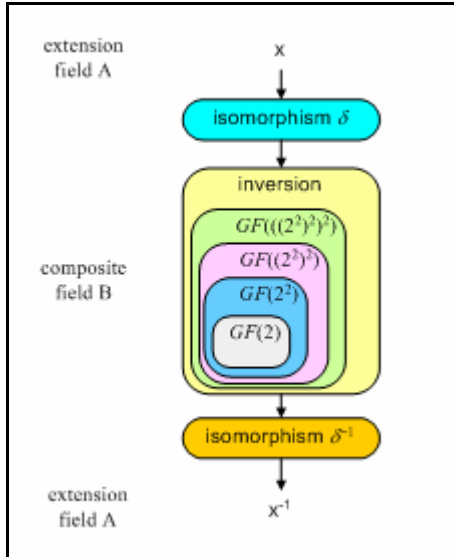


Figure 1-2. The basic idea of working in smaller field.

A single degree extension of $GF(2^8)$ is constructed with the composite field

$$\begin{cases} GF(2^4) : m(x) = x^4 + x + 1 \\ GF((2^4)^2) : m(x) = x^2 + x + \omega_{14} \end{cases}$$

where $\omega_{14} = [1001]_2$ and $m(x)$ is the corresponding irreducible polynomial, inversion in the extension field $GF(2^8)$ is performed through inversion in the composite field $GF((2^4)^2)$. The submitted design is done using the one that has been developed by Morioka and Satoh [4], from which we have chosen composite field $GF(((2^2)^2)^2)$. The later field is simply obtained by applying multiple extensions of smaller degrees.

Multiple extensions is done in the following:

$$\begin{cases} GF(2^2) : m(x) = x^2 + x + 1 \\ GF((2^2)^2) : m(x) = x^2 + x + \phi \\ GF(((2^2)^2)^2) : m(x) = x^2 + x + \chi \end{cases}$$

where $\phi = [10]_2$, $\chi = [1100]_2$. The correspondent composite field inverter is shown in Figure 1-3.

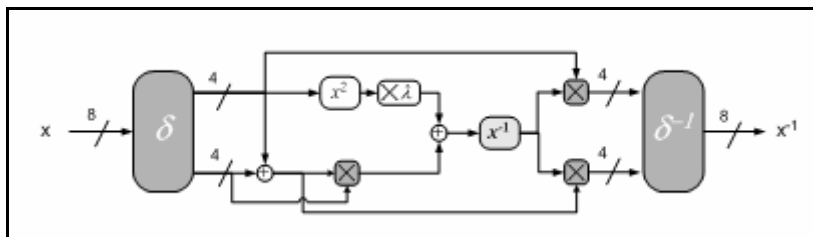


Figure 1-3. The composite field inverter showing 4-bits parallel processing.

Instead of explicitly stating the schematic (Figure 1-3) as separate components in HDL codes, the sub-components of the composite field inverter are converted in PPRM (Positive Polarity Reed-Muller) form: the pre-inversion section, the inversion section, and the post-inversion section [4]. As will be explained further in the following sections, we choose **3-stages PPRM** representation.

1.2.3. Design Project Hierarchy

All design entries were done in HDL Designer Series (FPGA Advantages 5.2). The project hierarchy in the view name is shown in Figure 1-4. The block diagram in Figure 1-1 is not coded as separate components, we use one entity instead.

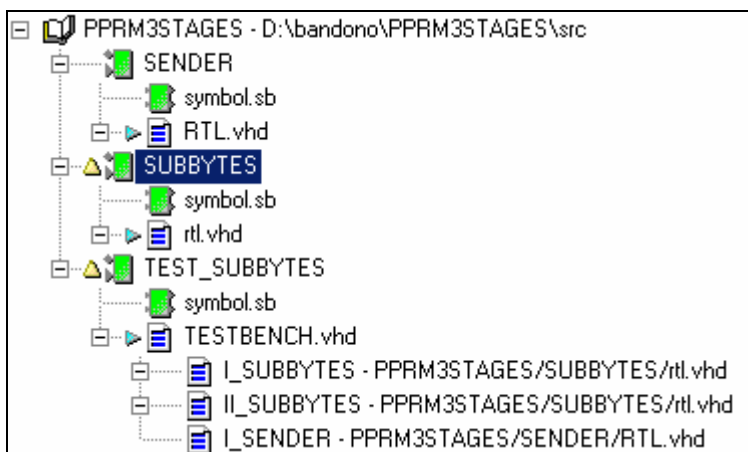


Figure 1-4. HDS browser view of the project library.

1.3. Appealing Point

We claim **originality** through the derivation of the stage representations (Section 2). We have derived back the composite field inverter, the corresponding isomorphism function, and the PPRM representations of the 3-stages. The results are pretty much identical to the ready-to-use versions that were appended in [4].

For evaluation purpose, we've synthesized also other S-Box designs. Since we haven't done proper documentations for each of the other S-Box, short explanations on how each was built are as follow:

- **Look Up-Table**

We simply modify the subbytes.vhd provided in the Design Contest 2004 homepage to have both SubBytes and InvSubBytes performable. It is basically a ROM.

- **Itoh and Tsujii**

The Design Contest site also contents example of $GF(2^8)$ inverter. The inversion circuit is based on Itoh Tsujii's algorithm where multiplicative inverse is calculated as $y^{-1} = y^{254}$. Implementation of the later 254 powering consists of multiplication circuit, $\wedge 2$ (power 2) circuit, $\wedge 4$ circuit, and $\wedge 16$ circuit all in $GF(2^8)$. Each of the powering circuit is obtained respectively one after another by deriving the multiplication circuit first. We always perform the necessary AND-XOR minimization during the process.

- **Composite Field $GF((2^4)^2)$**
 Basically, the same design approach as the submitted one is used. Hence, the following sections derivation-method holds. We use the isomorphism functions in [2] and the generic composite field inverter and constant in [4]. Four-bits basis operation is then performed over the subfield $GF(2^4)$.
- **4-stages PPRM**
 The only different to the 3-stages submitted is the staging.
- **SOP and POS**
 The circuit is obtained from its truth table by using two-level logic in terms of *sum of product* and *product of sum*.

1.3.1. Evaluation of Synthesized Circuit

Due to implementation purpose, the design is synthesized into *Xilinx FPGA XCV300PQ240* target library using Xilinx Foundation Series 3.1i . Other types of S-Box design are also synthesized. Comparisons on speed/area criterion are shown in Figure 1-5 and Table 1-2. Complete synthesis report of the submitted design can be found in Section 5.

The delay is normalized using the speed unit of the synthesized 50 input XOR. All synthesis reports are generated under the same constraints. Synthesis is optimized for speed (high effort) under 50 MHz clock constraint and is done using speed grade-6. The absolute values of delay/size can vary depending on the target libraries and synthesis tools, but in [5] the tests were showing almost the same circuit speed ratios.

Table 1-2. Comparison of synthesized circuits using Xilinx target device.

S-Box	XCV300PQ240	
	Size (gates)	Delay
Look Up-Table	2662	7.41
Itoh and Tsujii	1960	12.64
Composite Field $GF((2^4)^2)$	682	7.23
3-stages PPRM	979	8.53
4-stages PPRM	874	7.54
SOP	2410	6.63
POS	2602	7.50

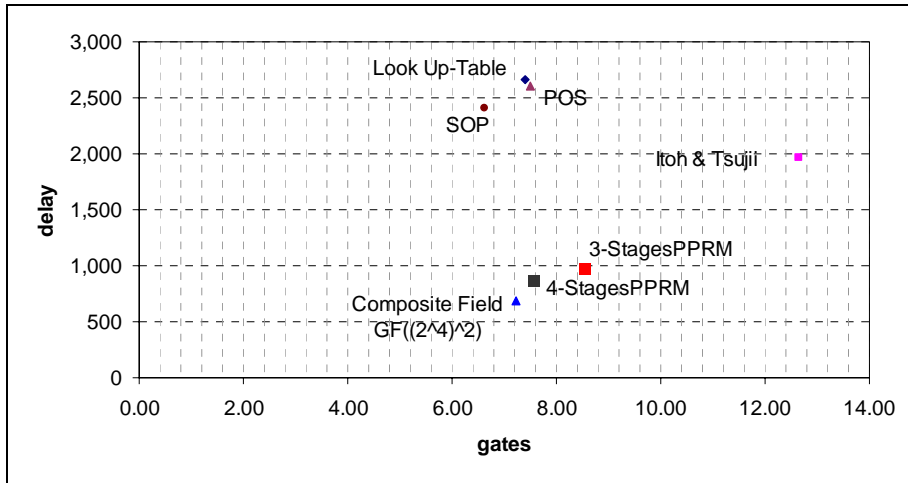


Figure 1-5. Graphical comparison.

1.3.2. Low Power Architecture

Logical AND-gates output '0' (low) for half of the possible input pairs. Due to this fact, logical AND-gates are 50% transparent to dynamic hazard. With similar meaning, XOR are 100% transparent to dynamic hazard (it outputs all the input hazards). Dynamic hazard occurs when different signal arrival times are different between multiple inputs of a gate. Serial connections between multiple gates and some of the internal gates generate hazards. When these hazards propagate along the circuit path, some extra power is consumed. Within the S-Box the gates can switch many times per single transition of the primary inputs.

By organizing AND array that feeds the primary input of XOR array in each stages of PPRM, the hazard transparency of the whole circuit is then reduced. Moreover, arranging in this fashion reduce the power consumption. The composite field inverter arrangement can be shown in Figure 1-6.

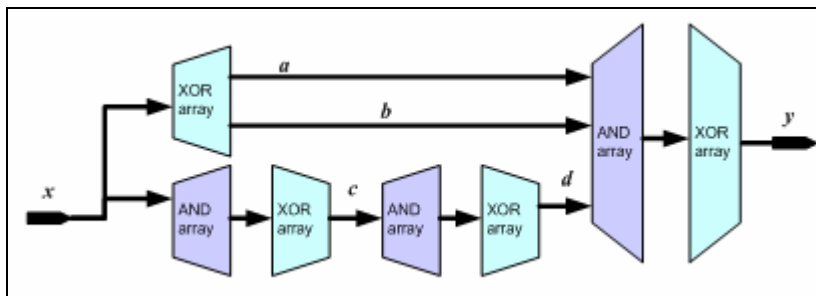


Figure 1-6. Staging in AND-XOR array view.

Full power analysis was done in [4] where in the method, a timing simulation at the gate level were performed using set of tests input data, and the switching activity of all internal gates were then logged. Power was computed from the simulation log and the cell information of the target ASIC library. Being much smaller S-Box, the multi-stage PPRM S-Box achieved the lowest power consumption of 29 μ W at 10 MHz using 0.13 μ m 1.5V technology. However, the submitted report isn't fully intended for power analysis. We only use the fact that we've arranged the inverter in similar fashion where the 3-stages had been shown in [4] to be the **lowest power consuming S-Box** though it wasn't the smallest n-stages.

2. Derivation of Stage Representations

As mathematical preliminaries in [3], basic operations over the Rijndael field will consist of multiplication, replaced with AND logic, and addition/subtraction, replaced with XOR logic. We derive back the composite field inverter circuit by using the same principals as were explained in [2] for $GF((2^4)^2)$. Then, staging is performed in this $GF(((2^2)^2)^2)$ inverter.

2.1. Derivation of the Composite Field Inversion Formula

The field $GF(2^8)$ can be viewed as $GF(((2^2)^2)^2)$. In both cases we have the same field, we just label the elements differently. We refer to $GF(2^8)$ as the *extension field*. The *extension field irreducible*, $m(x) = x^8 + x^4 + x^3 + x + 1$, defines addition and multiplication over that field. Subsequently, over the *composite field* $GF(((2^2)^2)^2)$, $m(x) = x^2 + x + \lambda$ had been chosen as the *composite field irreducible* in [4]. The coefficient λ is element of $GF((2^2)^2)$. As we continue to attempt computation in smaller field, the *subfield* $GF((2^2)^2)$ is then used. It has the *subfield irreducible* $m(x) = x^2 + x + \phi$, where ϕ is element of $GF(2^2)$. Our next attempt is to derive composite field inverter where we slice 8 bits input into 4 bits parallel processing. This is achievable because we only deal with 16 elements over the subfield $GF((2^2)^2)$.

If we label 256 elements of $GF(2^8)$ with a polynomial of degree 1 whose coefficients are in the subfield $GF((2^2)^2)$, we will have

$$\zeta \in GF(2^8) \Rightarrow \zeta = Ax + B$$

where $A, B \in GF((2^2)^2)$ (the subfield) and x is a root of the composite field irreducible $m(x) = x^2 + x + \lambda = 0$.

Suppose that $u, v \in GF((2^2)^2)$, where $u \bullet v = 1$, that is, $v = u^{-1}$. Through the above labeling we have

$$u = Ax + B$$

$$v = Cx + D$$

where $A, B, C, D \in GF((2^2)^2)$ and x is the same root as before. The multiplication (denoted by \bullet) over the composite field $GF(((2^2)^2)^2)$ is performed as

$$u(x) \bullet v(x) = 1$$

$$((Ax + B)(Cx + D)) \bmod(x^2 + x + \lambda) = 1$$

$$(ACx^2 + (AD + BC)x + BD) \bmod(x^2 + x + \lambda) = 1 \quad (1)$$

Having x as a root of $m(x) = x^2 + x + \lambda = 0$, thus the modulo can be computed using

$$x^2 + x + \lambda = 0$$

$$x^2 = x + \lambda \tag{2}$$

Substituting $x^2 = x + \lambda$ (2)(2) into (1) yields

$$AC(x + \lambda) + (AD + BC)x + BD = 1$$

$$(AC + AD + BC)x + (BD + AC\lambda) = 1$$

The left side polynomial equals to the result of multiplication $u(x) \cdot v(x)$ which represents an element of $GF((2^2)^2)$. Thus, equating will result

$$AC + AD + BC = 0 \tag{3}$$

$$BD + AC\lambda = 1 \tag{4}$$

To find C and D as functions of A and B , we can use both equation (3) and (4)

Rearrangement of equation (3) is

$$D = CA^{-1}(A + B) \tag{5}$$

We first find C as a function of A and B by substituting (5) into (4) which yields

$$B(CA^{-1}(A + B)) + AC\lambda = 1$$

$$C(B(A + B) + A^2\lambda) = A$$

$$C = A(AB + B^2 + A^2\lambda)^{-1} \tag{6}$$

D as a function of A and B can be found by substitution of equation (6) into (5). Thus,

$$D = (A + B)(AB + B^2 + A^2\lambda)^{-1} \tag{7}$$

The last two equations ((6) and (7)) can be implemented in hardware using the schematic in Fig.

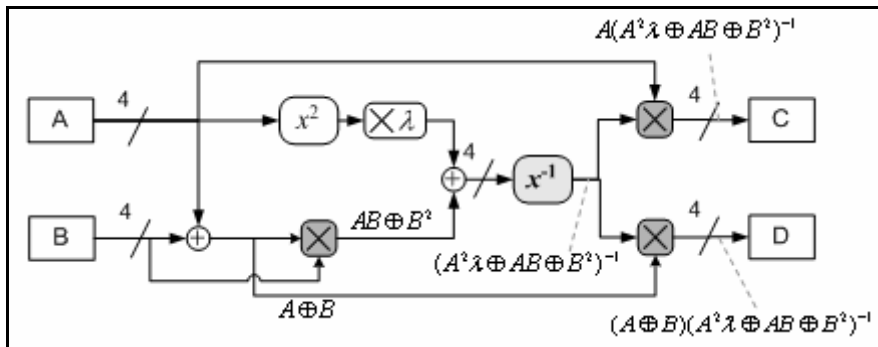


Figure 2-1. The basic composite field inverter.

2.2. Derivation of Isomorphism Function

An eight bit data $\tau = \tau_7\tau_6\tau_5\tau_4\tau_3\tau_2\tau_1\tau_0$ can be expressed as polynomials in Galois Field $GF(2^8)$ in the following representation:

$$\tau(x) = \tau_7 \cdot x^7 + \tau_6 \cdot x^6 + \tau_5 \cdot x^5 + \tau_4 \cdot x^4 + \tau_3 \cdot x^3 + \tau_2 \cdot x^2 + \tau_1 \cdot x + \tau_0$$

Similar to the previous section labeling, the above eight bits data in $GF(2^8)$ can be viewed in $GF(((2^2)^2)^2)$ as

$$\tau = a\gamma + b \quad (8)$$

where $a, b \in GF((2^2)^2)$ and γ is a root of the composite field irreducible $S(w)$.

Continuing the extension, any c in $GF((2^2)^2)$ (Galois Field with 16 elements) can be labeled into

$$c = \chi_1\beta + \chi_0 \quad (9)$$

where $\chi_0, \chi_1 \in GF(2^2)$ and β is a root of subfield irreducible $R(z)$. Afterwards, any d in $GF(2^2)$ can be written as

$$d = \delta_1\alpha + \delta_0$$

where $\delta_0, \delta_1 \in GF(2)$ (Galois Field with 2 elements) and α is a root of the subfield irreducible $Q(y)$.

Thus, in terms of α, β , and γ we get (8) extended as

$$\tau = ((a_3\alpha + a_2)\beta + (a_1\alpha + a_0))\gamma + ((b_3\alpha + b_2)\beta + (b_1\alpha + b_0))$$

$$\tau = a_0\gamma + a_1\gamma\alpha + a_2\gamma\beta + a_3\gamma\alpha\beta + b_0 + b_1\alpha + b_2\beta + b_3\alpha\beta$$

stated in vector notation as

$$\vec{\tau} = (1 + \vec{\alpha} + \vec{\beta} + \vec{\alpha}\vec{\beta} \quad \vec{\gamma} + \vec{\gamma}\vec{\alpha} + \vec{\gamma}\vec{\beta} + \vec{\gamma}\vec{\alpha}\vec{\beta}) \begin{pmatrix} \vec{b} \\ \vec{a} \end{pmatrix}$$

where $\vec{\tau} = (\tau_0 \quad \tau_1 \quad \dots \quad \tau_7)^T$, $\vec{a} = (a_0 \quad a_1 \quad a_2 \quad a_3)^T$, and $\vec{b} = (b_0 \quad b_1 \quad b_2 \quad b_3)^T$.

Note that $\alpha, \beta, \gamma \in GF(2^8)$, so we can write the above equation in the following expanded form:

$$\begin{pmatrix} \tau_0 \\ \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \\ \tau_7 \end{pmatrix} = \begin{pmatrix} 1 & \alpha_0 & \beta_0 & (\alpha\beta)_0 & \gamma_0 & (\gamma\alpha)_0 & (\gamma\beta)_0 & (\gamma\alpha\beta)_0 \\ 0 & \alpha_1 & \beta_1 & (\alpha\beta)_1 & \gamma_1 & (\gamma\alpha)_1 & (\gamma\beta)_1 & (\gamma\alpha\beta)_1 \\ 0 & \alpha_2 & \beta_2 & (\alpha\beta)_2 & \gamma_2 & (\gamma\alpha)_2 & (\gamma\beta)_2 & (\gamma\alpha\beta)_2 \\ 0 & \alpha_3 & \beta_3 & (\alpha\beta)_3 & \gamma_3 & (\gamma\alpha)_3 & (\gamma\beta)_3 & (\gamma\alpha\beta)_3 \\ 0 & \alpha_4 & \beta_4 & (\alpha\beta)_4 & \gamma_4 & (\gamma\alpha)_4 & (\gamma\beta)_4 & (\gamma\alpha\beta)_4 \\ 0 & \alpha_5 & \beta_5 & (\alpha\beta)_5 & \gamma_5 & (\gamma\alpha)_5 & (\gamma\beta)_5 & (\gamma\alpha\beta)_5 \\ 0 & \alpha_6 & \beta_6 & (\alpha\beta)_6 & \gamma_6 & (\gamma\alpha)_6 & (\gamma\beta)_6 & (\gamma\alpha\beta)_6 \\ 0 & \alpha_7 & \beta_7 & (\alpha\beta)_7 & \gamma_7 & (\gamma\alpha)_7 & (\gamma\beta)_7 & (\gamma\alpha\beta)_7 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

It is clear that the matrix above is the isomorphism function δ^{-1} . With $a, b \in GF((2^2)^2)$, we can map the computation result in 4 bits basis (a and b) back to the initial bit-width of τ by using δ^{-1} .

The irreducible polynomials mentioned earlier in this section are

$$\left. \begin{aligned} GF(2^8): P(x) &= x^8 + x^4 + x^3 + x + 1 \\ GF(2^2): Q(y) &= y^2 + y + 1 \\ GF((2^2)^2): R(z) &= z^2 + z + \phi \\ GF(((2^2)^2)^2): S(w) &= w^2 + w + \lambda \end{aligned} \right\} \quad (10)$$

where $\phi = [10]_2$ and $\chi = [1100]_2$. Values of the δ^{-1} matrix elements can be found by solving $Q(\alpha) = 0$, $R(\beta) = 0$, and $S(\gamma) = 0$, respectively:

$$\begin{aligned} Q(\alpha) &= 0 \\ \alpha^2 + \alpha + 1 &= 0 \\ \alpha^2 + \alpha &= 1 \\ \alpha_6 x^7 + (\alpha_3 \oplus \alpha_5 \oplus \alpha_6) x^6 + \alpha_6 x^5 + (\alpha_2 \oplus \alpha_7) x^4 + (\alpha_3 \oplus \alpha_4 \oplus \alpha_5 \oplus \alpha_6 \oplus \alpha_7) x^3 \\ &+ (\alpha_1 \oplus \alpha_2 \oplus \alpha_5) x^2 + (\alpha_1 \oplus \alpha_4 \oplus \alpha_6 \oplus \alpha_7) x + (\alpha_4 \oplus \alpha_6) = 1 \end{aligned}$$

The solution of the above equation is $\alpha = 1011110\alpha_0$. If we choose $\alpha_0 = 0$, then $\alpha = 10111100$ or in polynomial form $\alpha(x) = x^7 + x^5 + x^4 + x^3 + x^2$.

$$\begin{aligned} R(\beta) &= 0 \\ \beta^2 + \beta + \phi &= 0 \\ \beta^2 + \beta = \phi = \{10\}_2 = \alpha &= x^7 + x^5 + x^4 + x^3 + x^2 \\ \beta_6 x^7 + (\beta_3 \oplus \beta_5 \oplus \beta_6) x^6 + \beta_6 x^5 + (\beta_2 \oplus \beta_7) x^4 + (\beta_3 \oplus \beta_4 \oplus \beta_5 \oplus \beta_6 \oplus \beta_7) x^3 \\ &+ (\beta_1 \oplus \beta_2 \oplus \beta_5) x^2 + (\beta_1 \oplus \beta_4 \oplus \beta_6 \oplus \beta_7) x + (\beta_4 \oplus \beta_6) = x^7 + x^5 + x^4 + x^3 + x^2 \end{aligned}$$

The solution of the above equation is $\beta = 0101110\beta_0$. If we choose $\beta_0 = 1$, then $\beta = 01011101$ or in polynomial form $\beta(x) = x^6 + x^4 + x^3 + x^2 + 1$.

$$\begin{aligned} S(\gamma) &= 0 \\ \gamma^2 + \gamma + \lambda &= 0 \\ \gamma^2 + \gamma = \lambda = \{1100\}_2 = (\alpha + 1)\beta = \alpha\beta + \beta &= 01010001 \\ \gamma_6 x^7 + (\gamma_3 \oplus \gamma_5 \oplus \gamma_6) x^6 + \gamma_6 x^5 + (\gamma_2 \oplus \gamma_7) x^4 + (\gamma_3 \oplus \gamma_4 \oplus \gamma_5 \oplus \gamma_6 \oplus \gamma_7) x^3 \\ &+ (\gamma_1 \oplus \gamma_2 \oplus \gamma_5) x^2 + (\gamma_1 \oplus \gamma_4 \oplus \gamma_6 \oplus \gamma_7) x + (\gamma_4 \oplus \gamma_6) = x^5 + x^4 + 1 \end{aligned}$$

The solution of the above equation is $\gamma = 0001111\gamma_0$. If we choose $\gamma_0 = 1$, then $\gamma = 00011111$ or in polynomial form $\gamma(x) = x^5 + x^4 + x^3 + x^2 + x + 1$.

Hence, we have $\alpha = 10111100$, $\beta = 01011101$, and $\gamma = 00011111$. Respectively, $\alpha \cdot \beta = 00001100$, $\gamma \cdot \alpha = 10111011$, $\gamma \cdot \beta = 11110001$, and $\gamma \cdot \alpha \cdot \beta = 10000100$.

Inserting the values into the matrix results in

$$\delta^{-1} = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

From the above function, we can find the isomorphism function δ as

$$\delta = (\delta^{-1})^{-1} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

2.3. Derivation of 3-Stages PPRM Representations

The term *three-stages* refers to the amount of partition done for the composite field inverter. For the inverter which we have derived above, the partitioning is shown in Fig. Different staging would cost different speed/area as was shown in [4].

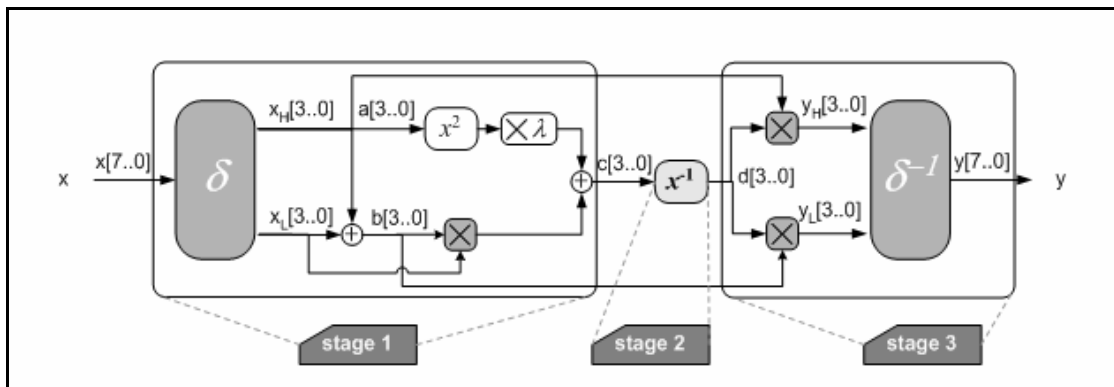


Figure 2-2. Staging of the composite field inverter.

The 8-bits input of the inverter are denoted x and the 8-bits output are denoted y . The other variables a , b , c , and d are the internal wires.

Eight-bits x are sliced into two 4-bits data x_H and x_L through isomorphism function δ . So, we obtain

$$a = x_H \text{ and } b = x_H \oplus x_L \quad (11)$$

In terms of x , x_H , and x_L , the matrix form is

$$\begin{pmatrix} x_H \\ x_L \end{pmatrix} = \delta \cdot x$$

or

$$\begin{pmatrix} x_{L0} \\ x_{L1} \\ x_{L2} \\ x_{L3} \\ x_{H0} \\ x_{H1} \\ x_{H2} \\ x_{H3} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{pmatrix}$$

Substituting for a and b in (10), we obtain

$$\left. \begin{aligned} a_3 = x_{H3} &= x_7 \oplus x_5 \\ a_2 = x_{H2} &= x_7 \oplus x_6 \oplus x_4 \oplus x_3 \oplus x_2 \oplus x_1 \\ a_1 = x_{H1} &= x_7 \oplus x_5 \oplus x_3 \oplus x_2 \\ a_0 = x_{H0} &= x_7 \oplus x_5 \oplus x_3 \oplus x_2 \oplus x_1 \\ b_3 = x_{H3} \oplus x_{L3} &= x_6 \oplus x_5 \oplus x_2 \oplus x_1 \\ b_2 = x_{H2} \oplus x_{L2} &= x_6 \\ b_1 = x_{H1} \oplus x_{L1} &= x_7 \oplus x_6 \oplus x_5 \oplus x_4 \oplus x_3 \oplus x_2 \oplus x_1 \\ b_0 = x_{H0} \oplus x_{L0} &= x_7 \oplus x_6 \oplus x_5 \oplus x_3 \oplus x_2 \oplus x_0 \end{aligned} \right\} \quad (12)$$

Through out this section, we are still referring to the list of irreducible polynomials and constants in (10). In the inverter Fig we have the first stage stated

$$c = x_H^2 \lambda \oplus x_H x_L \oplus x_L^2 = a^2 \lambda \oplus ab \oplus b^2 \quad (13)$$

With $m, n \in GF(2^2)$, $m = m_1 n_0$, and $n = n_1 n_0$, the multiplication $p = m \cdot n$ over the field is performed

$$\begin{aligned} p &= m \cdot n = (m_1 \alpha + m_0)(n_1 \alpha + n_0) \\ p &= m_1 n_1 \alpha^2 + (m_1 n_0 + m_0 n_1) \alpha + m_0 n_0 \end{aligned}$$

where $m_i, n_i \in GF(2)$ (binary) and $Q(\alpha) = 0$ which gives $\alpha^2 + \alpha + 1 = 0$. Hence, substituting for α^2 yields

$$\begin{aligned} p &= m_1 n_1 (\alpha + 1) + (m_1 n_0 + m_0 n_1) \alpha + m_0 n_0 \\ p &= (m_1 n_1 + m_1 n_0 + m_0 n_1) \alpha + (m_1 n_1 + m_0 n_0) \end{aligned}$$

With $p = p_1 p_0$, a more hardware-realistic view gives

$$\left. \begin{aligned} p_1 &= (m_1 \cdot n_1 \oplus m_1 \cdot n_0 \oplus m_0 \cdot n_1) \\ p_0 &= (m_1 \cdot n_1 \oplus m_0 \cdot n_0) \end{aligned} \right\} \quad (14)$$

where additions are denoted by \oplus (bitwise XOR) and binary multiplications are denoted by \cdot (AND logic).

Over $GF((2^2)^2)$, multiplication is performed in similar effort. Thus, for $a, b \in GF((2^2)^2)$, we have $q = a \cdot b$ computed

$$\begin{aligned} q &= a \cdot b = (a_H \beta + a_L)(b_H \beta + b_L) \\ q &= a_H b_H \beta^2 + (a_H b_L + a_L b_H) \beta + a_L b_L \end{aligned}$$

where $R(\beta) = \beta^2 + \beta + \phi = 0$ and $a_H, a_L, b_H, b_L \in GF(2^2)$. Hence, substituting for β^2 yields

$$\begin{aligned} q &= a_H b_H (\beta + \phi) + (a_H b_L + a_L b_H) \beta + a_L b_L \\ q &= (a_H b_H + a_H b_L + a_L b_H) \beta + (a_H b_H \phi + a_L b_L) \end{aligned}$$

where $a_H = a_3 \alpha + a_2$, $a_L = a_1 \alpha + a_0$, $b_H = b_3 \alpha + b_2$, $b_L = b_1 \alpha + b_0$ and $q = (q_3 \alpha + q_2) \beta + (q_1 \alpha + q_0)$. All the multiplication terms in $a_i b_i$ above are over $GF(2^2)$.

From the previous multiplication over $GF(2^2)$ and having $\phi = [10]_2$, we obtain

$$\begin{aligned} q_3 &= a_3 \cdot b_3 \oplus a_3 \cdot b_2 \oplus a_3 \cdot b_1 \oplus a_3 \cdot b_0 \oplus a_2 \cdot b_3 \oplus a_2 \cdot b_1 \oplus a_1 \cdot b_3 \oplus a_1 \cdot b_2 \oplus a_0 \cdot b_3 \\ q_2 &= a_3 \cdot b_3 \oplus a_3 \cdot b_1 \oplus a_2 \cdot b_2 \oplus a_2 \cdot b_0 \oplus a_1 \cdot b_3 \oplus a_0 \cdot b_2 \\ q_1 &= a_3 \cdot b_2 \oplus a_2 \cdot b_3 \oplus a_2 \cdot b_2 \oplus a_1 \cdot b_1 \oplus a_1 \cdot b_0 \oplus a_0 \cdot b_1 \\ q_0 &= a_3 \cdot b_3 \oplus a_3 \cdot b_2 \oplus a_2 \cdot b_3 \oplus a_1 \cdot b_1 \oplus a_0 \cdot b_0 \end{aligned}$$

The above derivation leads to the stage representation of (13) where we deal with operation over $GF((2^2)^2)$. For any $r = a^2$ in $GF((2^2)^2)$, we have

$$\begin{aligned} r_3 &= a_3 \\ r_2 &= a_3 \oplus a_2 \\ r_1 &= a_2 \oplus a_1 \\ r_0 &= a_3 \oplus a_1 \oplus a_0 \end{aligned}$$

Thus, we obtain (13) in

$$\left. \begin{aligned}
 c_3 &= a_2 \oplus a_1 \oplus a_0 \oplus b_3 \oplus a_3 \cdot b_3 \oplus a_3 \cdot b_2 \oplus a_3 \cdot b_1 \oplus a_3 \cdot b_0 \oplus a_2 \cdot b_3 \\
 &\quad \oplus a_2 \cdot b_1 \oplus a_1 \cdot b_3 \oplus a_1 \cdot b_2 \oplus a_0 \cdot b_3 \\
 c_2 &= a_3 \oplus a_0 \oplus b_3 \oplus b_2 \oplus a_3 \cdot b_3 \oplus a_3 \cdot b_1 \oplus a_2 \cdot b_2 \oplus a_2 \cdot b_0 \oplus a_1 \cdot b_3 \\
 &\quad \oplus a_0 \cdot b_2 \\
 c_1 &= a_3 \oplus b_2 \oplus b_1 \oplus a_3 \cdot b_2 \oplus a_2 \cdot b_3 \oplus a_2 \cdot b_2 \oplus a_1 \cdot b_1 \oplus a_1 \cdot b_0 \oplus a_0 \cdot b_1 \\
 c_0 &= a_3 \oplus a_2 \oplus b_3 \oplus b_1 \oplus b_0 \oplus a_3 \cdot b_3 \oplus a_3 \cdot b_2 \oplus a_2 \cdot b_3 \oplus a_1 \cdot b_1 \oplus a_0 \cdot b_0
 \end{aligned} \right\} (15)$$

Finally, the stage 1 representation is obtained by substituting (12) into (15).

$$\begin{aligned}
 c_3 &= x_7 \cdot x_5 \oplus x_7 \cdot x_4 \oplus x_7 \cdot x_1 \oplus x_7 \cdot x_0 \oplus x_6 \cdot x_5 \oplus x_6 \cdot x_4 \oplus x_6 \cdot x_2 \oplus x_6 \cdot x_1 \\
 &\quad \oplus x_5 \cdot x_4 \oplus x_5 \cdot x_2 \oplus x_5 \cdot x_1 \oplus x_5 \cdot x_0 \oplus x_4 \cdot x_2 \oplus x_4 \cdot x_1 \oplus x_3 \cdot x_2 \oplus x_3 \cdot x_1 \oplus x_2 \cdot x_1 \\
 c_2 &= x_7 \cdot x_6 \oplus x_7 \cdot x_5 \oplus x_7 \cdot x_3 \oplus x_7 \cdot x_2 \oplus x_7 \cdot x_0 \oplus x_6 \cdot x_5 \oplus x_6 \cdot x_3 \oplus x_6 \cdot x_2 \oplus x_6 \cdot x_1 \\
 &\quad \oplus x_6 \cdot x_0 \oplus x_5 \cdot x_3 \oplus x_5 \cdot x_2 \oplus x_4 \cdot x_3 \oplus x_4 \cdot x_2 \oplus x_4 \cdot x_0 \oplus x_3 \cdot x_2 \oplus x_3 \cdot x_0 \\
 &\quad \oplus x_2 \cdot x_0 \oplus x_1 \cdot x_0 \\
 c_1 &= x_7 \cdot x_5 \oplus x_7 \cdot x_2 \oplus x_7 \cdot x_0 \oplus x_6 \cdot x_5 \oplus x_6 \cdot x_3 \oplus x_5 \cdot x_4 \oplus x_5 \cdot x_3 \oplus x_5 \cdot x_2 \oplus x_5 \cdot x_0 \\
 &\quad \oplus x_4 \cdot x_2 \oplus x_3 \cdot x_2 \oplus x_3 \cdot x_0 \oplus x_2 \cdot x_1 \oplus x_2 \cdot x_0 \oplus x_7 \oplus x_5 \oplus x_4 \oplus x_2 \oplus x_1 \\
 c_0 &= x_7 \cdot x_6 \oplus x_7 \cdot x_4 \oplus x_7 \cdot x_0 \oplus x_6 \cdot x_5 \oplus x_6 \cdot x_4 \oplus x_6 \cdot x_3 \oplus x_6 \cdot x_1 \oplus x_5 \cdot x_3 \oplus x_5 \cdot x_0 \\
 &\quad \oplus x_4 \cdot x_3 \oplus x_4 \cdot x_1 \oplus x_3 \cdot x_2 \oplus x_3 \cdot x_1 \oplus x_3 \cdot x_0 \oplus x_2 \cdot x_0 \oplus x_1 \cdot x_0 \oplus x_6 \oplus x_5 \\
 &\quad \oplus x_3 \oplus x_2 \oplus x_0
 \end{aligned}$$

In stage 3, we just apply $y_H = a \cdot d$ and $y_L = d \cdot b$, both over the subfield $GF((2^2)^2)$.

Applying isomorphism function δ^{-1} as $y = \delta^{-1}(y_L \quad y_H)^T$, then we obtain

$$\begin{aligned}
 y_7 &= a_3 \cdot d_0 \oplus a_2 \cdot d_1 \oplus a_2 \cdot d_0 \oplus a_1 \cdot d_2 \oplus a_1 \cdot d_1 \oplus a_1 \cdot d_0 \oplus a_0 \cdot d_3 \oplus a_0 \cdot d_2 \oplus a_0 \cdot d_1 \oplus \\
 &\quad d_3 \cdot b_2 \oplus d_2 \cdot b_3 \oplus d_2 \cdot b_2 \oplus d_1 \cdot b_1 \oplus d_1 \cdot b_0 \oplus d_0 \cdot b_1 \\
 y_6 &= a_3 \cdot d_3 \oplus a_3 \cdot d_1 \oplus a_2 \cdot d_2 \oplus a_2 \cdot d_0 \oplus a_1 \cdot d_3 \oplus a_0 \cdot d_2 \oplus d_3 \cdot b_3 \oplus d_3 \cdot b_1 \oplus d_2 \cdot b_2 \oplus \\
 &\quad d_2 \cdot b_0 \oplus d_1 \cdot b_3 \oplus d_0 \cdot b_2 \\
 y_5 &= a_3 \cdot d_3 \oplus a_3 \cdot d_2 \oplus a_3 \cdot d_1 \oplus a_2 \cdot d_3 \oplus a_2 \cdot d_0 \oplus a_1 \cdot d_3 \oplus a_1 \cdot d_1 \oplus a_1 \cdot d_0 \oplus a_0 \cdot d_2 \oplus \\
 &\quad a_0 \cdot d_1 \oplus d_3 \cdot b_2 \oplus d_2 \cdot b_3 \oplus d_2 \cdot b_2 \oplus d_1 \cdot b_1 \oplus d_1 \cdot b_0 \oplus d_0 \cdot b_1 \\
 y_4 &= a_3 \cdot d_1 \oplus a_2 \cdot d_0 \oplus a_1 \cdot d_3 \oplus a_1 \cdot d_0 \oplus a_0 \cdot d_2 \oplus a_0 \cdot d_1 \oplus a_0 \cdot d_0 \oplus d_3 \cdot b_3 \oplus d_3 \cdot b_2 \oplus \\
 &\quad d_3 \cdot b_1 \oplus d_2 \cdot b_3 \oplus d_2 \cdot b_0 \oplus d_1 \cdot b_3 \oplus d_1 \cdot b_1 \oplus d_1 \cdot b_0 \oplus d_0 \cdot b_2 \oplus d_0 \cdot b_1 \\
 y_3 &= a_3 \cdot d_3 \oplus a_2 \cdot d_2 \oplus a_1 \cdot d_0 \oplus a_0 \cdot d_1 \oplus a_0 \cdot d_0 \oplus d_3 \cdot b_0 \oplus d_2 \cdot b_1 \oplus d_2 \cdot b_0 \oplus d_1 \cdot b_2 \oplus \\
 &\quad d_1 \cdot b_1 \oplus d_1 \cdot b_0 \oplus d_0 \cdot b_3 \oplus d_0 \cdot b_2 \oplus d_0 \cdot b_1 \\
 y_2 &= a_3 \cdot d_1 \oplus a_3 \cdot d_0 \oplus a_2 \cdot d_1 \oplus a_1 \cdot d_3 \oplus a_1 \cdot d_2 \oplus a_1 \cdot d_1 \oplus a_0 \cdot d_3 \oplus a_0 \cdot d_0 \oplus d_3 \cdot b_0 \oplus \\
 &\quad d_2 \cdot b_1 \oplus d_2 \cdot b_0 \oplus d_1 \cdot b_2 \oplus d_1 \cdot b_1 \oplus d_1 \cdot b_0 \oplus d_0 \cdot b_3 \oplus d_0 \cdot b_2 \oplus d_0 \cdot b_1 \\
 y_1 &= a_3 \cdot d_3 \oplus a_2 \cdot d_2 \oplus a_1 \cdot d_0 \oplus a_0 \cdot d_1 \oplus a_0 \cdot d_0 \\
 y_0 &= a_3 \cdot d_1 \oplus a_2 \cdot d_0 \oplus a_1 \cdot d_3 \oplus a_1 \cdot d_0 \oplus a_0 \cdot d_2 \oplus a_0 \cdot d_1 \oplus a_0 \cdot d_0 \oplus d_3 \cdot b_2 \oplus d_3 \cdot b_1 \oplus \\
 &\quad d_2 \cdot b_3 \oplus d_2 \cdot b_2 \oplus d_2 \cdot b_0 \oplus d_1 \cdot b_3 \oplus d_1 \cdot b_1 \oplus d_0 \cdot b_2 \oplus d_0 \cdot b_0
 \end{aligned}$$

Stage 2 of the composite field inverter is an inverter as well. It computes multiplicative inverse over the subfield $GF((2^2)^2)$. The structure derived in Section 2.1 still holds for the

subfield inverter. We only change the labeling of the 256 elements of $GF(((2^2)^2)^2)$. Alternatively, all 16 elements of the subfield $GF((2^2)^2)$ are labeled in the same manner as done in (9). By changing all the label coefficients and the irreducible polynomial constant with elements of $GF(2^2)$, we get the schematic in fig.

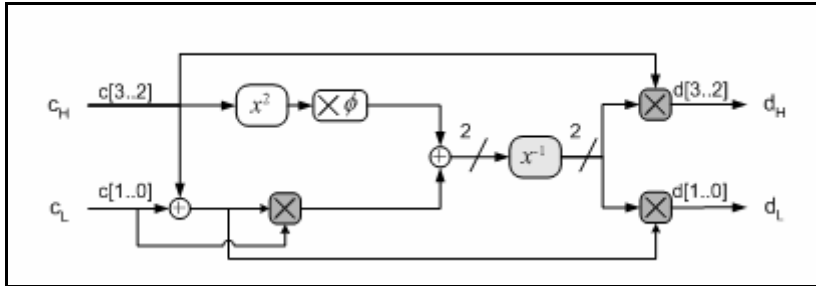


Figure 2-3. Stage 2, the subfield $GF((2^2)^2)$ inverter.

From fig, we can express d as function of c which gives

$$\left. \begin{aligned} d_H &= c_H (c_H^2 \phi + c_H c_L + c_L^2)^{-1} \\ d_L &= (c_H + c_L) (c_H^2 \phi + c_H c_L + c_L^2)^{-1} \end{aligned} \right\} \quad (16)$$

where $d_H = d_3\alpha + d_2$, $d_L = d_1\alpha + d_0$, $c_H = c_3\alpha + c_2$, and $c_L = c_1\alpha + c_0$. Since the computations are over $GF(2^2)$, we can reuse the multiplication in (14), where applying all possible inputs yields

n_1, n_0	00	01	11	10
m_1, m_0	00	00	00	00
01	00	01	10	11
11	00	10	11	01
10	00	11	01	10

$p=mn$

Now, our objective is to derive the inverter for the subfield $GF(2^2)$ which is a component of the $GF((2^2)^2)$ inverter (Figure 2-3). Rearrangement of the previous K-map gives

m_1	0	1
m_0	0	1
0	00	11
1	01	10

$n = m^{-1}$

Thus, for the $n = m^{-1}$ K-map, minimization results are $n_1 = m_1$ and $n_0 = m_1 \oplus m_0$. By applying the later multiplicative inverse and operations over $GF(2^2)$ to (16), we obtain the stage 2 representation as

$$d_3 = (c_3 \cdot c_2 \cdot c_1) \oplus (c_3 \cdot c_0) \oplus c_3 \oplus c_2$$

$$d_2 = (c_3 \cdot c_2 \cdot c_1) \oplus (c_3 \cdot c_2 \cdot c_0) \oplus (c_3 \cdot c_0) \oplus (c_2 \cdot c_1) \oplus c_2$$

$$d_1 = (c_3 \cdot c_2 \cdot c_1) \oplus (c_3 \cdot c_1 \cdot c_0) \oplus (c_2 \cdot c_0) \oplus c_3 \oplus c_2 \oplus c_1$$

$$d_0 = (c_3 \cdot c_2 \cdot c_1) \oplus (c_3 \cdot c_2 \cdot c_0) \oplus (c_3 \cdot c_1 \cdot c_0) \oplus (c_3 \cdot c_1) \oplus (c_3 \cdot c_0) \oplus (c_2 \cdot c_1 \cdot c_0) \\ \oplus (c_2 \cdot c_1) \oplus c_2 \oplus c_1 \oplus c_0$$

3. VHDL Listings

3.1. The SubBytes HDL

```
--
-- File Name : SubBytes.vhd
-- Function : combinational logic of SubBytes transform using pprm 3
stages
--
--
-- Created by - Micro Team
--

LIBRARY ieee;
USE ieee.std_logic_1164.all;
USE ieee.numeric_std.all;

ENTITY SubBytes IS
-- Declarations
  port (CLK      : in  std_logic;
        RESET    : in  std_logic;
        XIN      : in  unsigned( 7 downto 0);
        INV      : in  std_logic;
        YOUT     : out unsigned( 7 downto 0)
  );

END SubBytes ;

-- hds interface_end
ARCHITECTURE rtl OF SubBytes IS

  signal invin : unsigned(7 downto 0);
  signal invout : unsigned(7 downto 0);

  Function Inverse_GF(X: unsigned(7 downto 0)) Return unsigned IS

    Variable A : unsigned(3 downto 0);
    Variable B : unsigned(3 downto 0);
    Variable C : unsigned(3 downto 0);
    Variable D : unsigned(3 downto 0);
    Variable Y : unsigned(7 downto 0);

  Begin

    -- STAGE 1 --

    --Define A--
    A(3) := X(7) xor X(5);
    A(2) := X(7) xor X(6) xor X(4) xor X(3) xor X(2) xor X(1);
    A(1) := X(7) xor X(5) xor X(3) xor X(2);
    A(0) := X(7) xor X(5) xor X(3) xor X(2) xor X(1);

    --Define B--
    B(3) := X(6) xor X(5) xor X(2) xor X(1);
    B(2) := X(6);
```

```
B(1) := X(7) xor X(6) xor X(5) xor X(4) xor X(3) xor X(2) xor X(1);
B(0) := X(7) xor X(6) xor X(5) xor X(3) xor X(2) xor X(0);

--Define C--
C(3) := (X(7) and X(5)) xor (X(7) and X(4)) xor (X(7) and X(1)) xor
(X(7) and X(0)) xor (X(6) and X(5)) xor (X(6) and X(4))
xor (X(6) and X(2)) xor (X(6) and X(1)) xor (X(5) and X(4)) xor
(X(5) and X(2)) xor (X(5) and X(1)) xor (X(5) and X(0))
xor (X(4) and X(2)) xor (X(4) and X(1)) xor (X(3) and X(2)) xor
(X(3) and X(1)) xor (X(2) and X(1)) ;

C(2) := (X(7) and X(6)) xor (X(7) and X(5)) xor (X(7) and X(3)) xor
(X(7) and X(2)) xor (X(7) and X(0)) xor (X(6) and X(5))
xor (X(6) and X(3)) xor (X(6) and X(2)) xor (X(6) and X(1)) xor
(X(6) and X(0)) xor (X(5) and X(3)) xor (X(5) and X(2))
xor (X(4) and X(3)) xor (X(4) and X(2)) xor (X(4) and X(0)) xor
(X(3) and X(2)) xor (X(3) and X(0)) xor (X(2) and X(0))
xor (X(1) and X(0));

C(1) := (X(7) and X(5)) xor (X(7) and X(2)) xor (X(7) and X(0)) xor
(X(6) and X(5)) xor (X(6) and X(3)) xor (X(5) and X(4))
xor (X(5) and X(3)) xor (X(5) and X(2)) xor (X(5) and X(0)) xor
(X(4) and X(2)) xor (X(3) and X(2)) xor (X(3) and X(0))
xor (X(2) and X(1)) xor (X(2) and X(0)) xor X(7) xor X(5) xor X(4)
xor X(2) xor X(1);

C(0) := (X(7) and X(6)) xor (X(7) and X(4)) xor (X(7) and X(0)) xor
(X(6) and X(5)) xor (X(6) and X(4)) xor (X(6) and X(3))
xor (X(6) and X(1)) xor (X(5) and X(3)) xor (X(5) and X(0)) xor
(X(4) and X(3)) xor (X(4) and X(1)) xor (X(3) and X(2))
xor (X(3) and X(1)) xor (X(3) and X(0)) xor (X(2) and X(0)) xor
(X(1) and X(0))
xor X(6) xor X(5) xor X(3) xor X(2) xor X(0);

-- STAGE 2 --

--Define D--
D(3) := (C(3) and C(2) and C(1)) xor (C(3) and C(0)) xor C(3) xor
C(2);
D(2) := (C(3) and C(2) and C(1)) xor (C(3) and C(2) and C(0)) xor
(C(3) and C(0)) xor (C(2) and C(1)) xor C(2);
D(1) := (C(3) and C(2) and C(1)) xor (C(3) and C(1) and C(0)) xor
(C(2) and C(0)) xor C(3) xor C(2) xor C(1);
D(0) := (C(3) and C(2) and C(1)) xor (C(3) and C(2) and C(0)) xor
(C(3) and C(1) and C(0)) xor (C(3) and C(1))
xor (C(3) and C(0)) xor (C(2) and C(1) and C(0)) xor (C(2) and C(1))
xor C(2) xor C(1) xor C(0);

-- STAGE 3 --

--Define Y--
Y(7) := (A(3) and D(0)) xor (A(2) and D(1)) xor (A(2) and D(0)) xor
(A(1) and D(2)) xor (A(1) and D(1))
xor (A(1) and D(0)) xor (A(0) and D(3)) xor (A(0) and D(2)) xor
(A(0) and D(1)) xor (D(3) and B(2))
xor (D(2) and B(3)) xor (D(2) and B(2)) xor (D(1) and B(1)) xor
(D(1) and B(0)) xor (D(0) and B(1));
```

```
Y(6) := (A(3) and D(3)) xor (A(3) and D(1)) xor (A(2) and D(2)) xor
(A(2) and D(0)) xor (A(1) and D(3))
xor (A(0) and D(2)) xor (D(3) and B(3)) xor (D(3) and B(1)) xor
(D(2) and B(2)) xor (D(2) and B(0))
xor (D(1) and B(3)) xor (D(0) and B(2));
```

```
Y(5) := (A(3) and D(3)) xor (A(3) and D(2)) xor (A(3) and D(1)) xor
(A(2) and D(3)) xor (A(2) and D(0))
xor (A(1) and D(3)) xor (A(1) and D(1)) xor (A(1) and D(0)) xor
(A(0) and D(2)) xor (A(0) and D(1))
xor (D(3) and B(2)) xor (D(2) and B(3)) xor (D(2) and B(2)) xor
(D(1) and B(1)) xor (D(1) and B(0))
xor (D(0) and B(1));
```

```
Y(4) := (A(3) and D(1)) xor (A(2) and D(0)) xor (A(1) and D(3)) xor
(A(1) and D(0)) xor (A(0) and D(2))
xor (A(0) and D(1)) xor (A(0) and D(0)) xor (D(3) and B(3)) xor
(D(3) and B(2)) xor (D(3) and B(1))
xor (D(2) and B(3)) xor (D(2) and B(0)) xor (D(1) and B(3)) xor
(D(1) and B(1)) xor (D(1) and B(0))
xor (D(0) and B(2)) xor (D(0) and B(1));
```

```
Y(3) := (A(3) and D(3)) xor (A(2) and D(2)) xor (A(1) and D(0)) xor
(A(0) and D(1)) xor (A(0) and D(0))
xor (D(3) and B(0)) xor (D(2) and B(1)) xor (D(2) and B(0)) xor
(D(1) and B(2)) xor (D(1) and B(1))
xor (D(1) and B(0)) xor (D(0) and B(3)) xor (D(0) and B(2)) xor
(D(0) and B(1));
```

```
Y(2) := (A(3) and D(1)) xor (A(3) and D(0)) xor (A(2) and D(1)) xor
(A(1) and D(3)) xor (A(1) and D(2))
xor (A(1) and D(1)) xor (A(0) and D(3)) xor (A(0) and D(0)) xor
(D(3) and B(0)) xor (D(2) and B(1))
xor (D(2) and B(0)) xor (D(1) and B(2)) xor (D(1) and B(1)) xor
(D(1) and B(0)) xor (D(0) and B(3))
xor (D(0) and B(2)) xor (D(0) and B(1));
```

```
Y(1) := (A(3) and D(3)) xor (A(2) and D(2)) xor (A(1) and D(0)) xor
(A(0) and D(1)) xor (A(0) and D(0));
```

```
Y(0) := (A(3) and D(1)) xor (A(2) and D(0)) xor (A(1) and D(3)) xor
(A(1) and D(0)) xor (A(0) and D(2))
xor (A(0) and D(1)) xor (A(0) and D(0)) xor (D(3) and B(2)) xor
(D(3) and B(1)) xor (D(2) and B(3))
xor (D(2) and B(2)) xor (D(2) and B(0)) xor (D(1) and B(3)) xor
(D(1) and B(1)) xor (D(0) and B(2))
xor (D(0) and B(0));
```

```
return Y;
end inverse_GF;
```

```
BEGIN
```

```
INVERSE: process(CLK)
begin
    if rising_edge(CLK) then
        if (RESET='1') then
            invout <= "00000000";
```

```
        else
            invout <= Inverse_GF(invin);
        end if;
    end if;
end process INVERSE;

process (inv,xin,invout)
begin

    if(inv='0')then
        invin <= xin;

        yout(7) <= invout(7) xor invout(6) xor invout(5) xor invout(4) xor
invout(3) xor '0';
        yout(6) <= invout(6) xor invout(5) xor invout(4) xor invout(3) xor
invout(2) xor '1';
        yout(5) <= invout(5) xor invout(4) xor invout(3) xor invout(2) xor
invout(1) xor '1';
        yout(4) <= invout(4) xor invout(3) xor invout(2) xor invout(1) xor
invout(0) xor '0';
        yout(3) <= invout(7) xor invout(3) xor invout(2) xor invout(1) xor
invout(0) xor '0';
        yout(2) <= invout(7) xor invout(6) xor invout(2) xor invout(1) xor
invout(0) xor '0';
        yout(1) <= invout(7) xor invout(6) xor invout(5) xor invout(1) xor
invout(0) xor '1';
        yout(0) <= invout(7) xor invout(6) xor invout(5) xor invout(4) xor
invout(0) xor '1';
    else
        invin(7) <= xin(6) xor xin(4) xor xin(1) xor '0';
        invin(6) <= xin(5) xor xin(3) xor xin(0) xor '0';
        invin(5) <= xin(7) xor xin(4) xor xin(2) xor '0';
        invin(4) <= xin(6) xor xin(3) xor xin(1) xor '0';
        invin(3) <= xin(5) xor xin(2) xor xin(0) xor '0';
        invin(2) <= xin(7) xor xin(4) xor xin(1) xor '1';
        invin(1) <= xin(6) xor xin(3) xor xin(0) xor '0';
        invin(0) <= xin(7) xor xin(5) xor xin(2) xor '1';

        yout <= invout;

    end if;
end process;

END rtl;
```

3.2. Testbench HDL

```
-- Univ. of the Ryukyus LSI design contest 2004
-- SubBytes Transform Circuit for AES Cipher
-- file: test_subbytes.vhd
-- TESTBENCH
-- Tom Wada 2003/September/15
-- Modified by Micro Team

library IEEE;
use IEEE.STD_LOGIC_1164.all, IEEE.NUMERIC_STD.all;

entity TEST_SUBBYTES is
```

```
end TEST_SUBBYTES;

architecture TESTBENCH of TEST_SUBBYTES is
  -- sender
  component SENDER
    port ( CLK      : in  std_logic;
          RESET    : in  std_logic;
          PLAIN     : out unsigned (7 downto 0) );
  end component;
  -- subbytes
  component SUBBYTES
    port ( CLK      : in  std_logic;
          RESET     : in  std_logic;
          XIN       : in  unsigned(7 downto 0);
          INV       : in  std_logic;
          YOUT      : out unsigned(7 downto 0) );
  end component;

  -- system clock
  signal CLK : std_logic := '0' ;
  -- system reset
  signal RESET : std_logic := '1';
  -- cycle count
  signal cycle : integer :=0;
  -- wires on the board
  signal PLAIN,PLAIN2 : unsigned (7 downto 0);
  signal YOUT      : unsigned (7 downto 0);
  signal INV       : std_logic := '0';
  signal NOTINV   : std_logic := '1';

begin

  -- clock generator
  CLOCK_GEN: process
  begin
    if (cycle < 1000) then
      cycle <= cycle + 1;
      wait for 10 ns;
      CLK <= not CLK;
    else wait;
    end if;
  end process CLOCK_GEN;

  -- reset sequence
  RESET_GEN: process
  begin
    LOOP1: for N in 0 to 5 loop
      wait until falling_edge(CLK);
    end loop LOOP1;
    RESET <= '0';
  end process RESET_GEN;

  -- sender instance
  I_SENDER: SENDER port map(CLK,RESET,PLAIN);

  -- subbytes instance
  I_SUBBYTES: SUBBYTES port map(CLK,RESET,PLAIN,INV,YOUT);
  II_SUBBYTES: SUBBYTES port map(CLK,RESET,YOUT,NOTINV,PLAIN2);

end TESTBENCH;
```

3.3. The Sender HDL

```
-- hds header_start
-- hds header_end
-- Univ. of the Ryukyus LSI design contest 2004
-- SubBytes Transform Circuit for AES Cipher
-- file: sender.vhd
-- Sender generates 8 bit integer from 0 to 255
-- Tom Wada 2003/September/15

library IEEE;
use IEEE.STD_LOGIC_1164.all, IEEE.NUMERIC_STD.all;

entity SENDER is
    port ( CLK      : in  std_logic;
          RESET    : in  std_logic;
          PLAIN    : out unsigned (7 downto 0) );
end SENDER;

-- hds interface_end

architecture RTL of SENDER is
-- signal define
    signal count : unsigned (7 downto 0); -- 8 bit counter

begin
-----
-- 8 bit counter
-----
    COUNTER: process(CLK)
    begin
        if rising_edge(CLK) then
            if (RESET='1') then
                count <= "00000000";
            else
                count <= count + 1;
            end if;
        end if;
    end process COUNTER;

-----
-- OUTPUT GEN
-----
    PLAIN <= count;

end RTL;
```


4. Simulation

4.1. Testbench Circuit

Simulation is done using the provided testbench, `sender.vhd`. The SubBytes simulation result is verified using the S-Box table given in [3]. We slightly modify the testbench to have both SubByte and InvSubByte simulated.

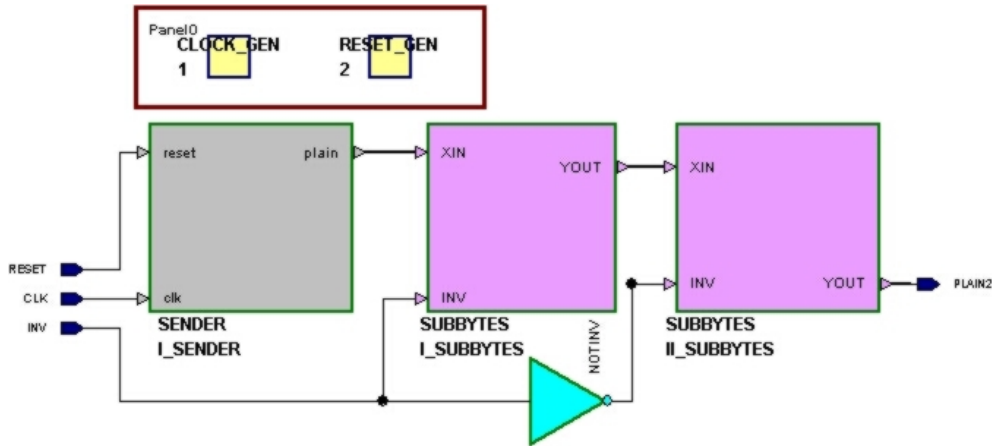
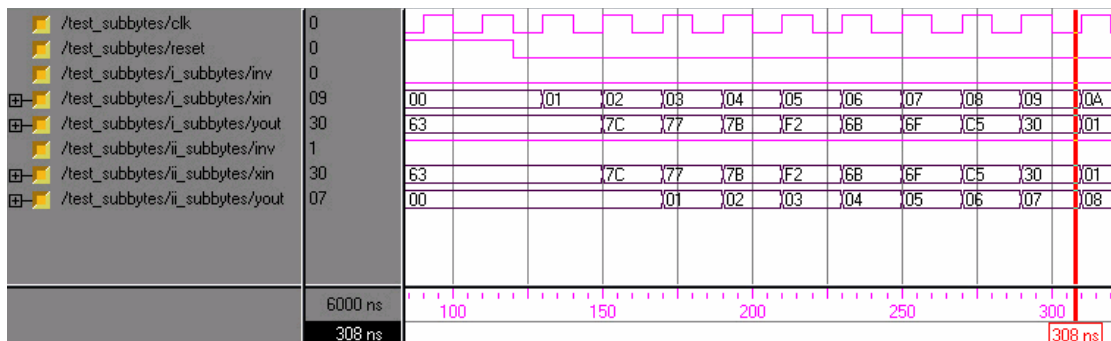


Figure 4-1. Testbench circuit for both S-Box and S-Box⁻¹ mode.

4.2. Waveform

The figure below is a screen capture of the simulated waveform. The complete simulation waveform can be found in the Appendix. The timing specification differs from the Level 1 requirement. This is because we only have the data path registered once. However the throughput is still the same.



5. Critical Path and Circuit Area

5.1. Timing Report of *parity.vhd*

```

=====
Timing constraint: Default path analysis
 50 items analyzed, 0 timing errors detected.
 Maximum delay is 15.997ns.
-----
Delay:      15.997ns A<3> to Y

Path A<3> to Y contains 5 levels of logic:
Path starting from Comp: P67.PAD
To          Delay type          Delay(ns)  Physical Resource
Resource(s)                                     Logical
-----
P67.I       Tiopi                       0.768R    A<3>
                                         A<3>.PAD
                                         C_A<3>
CLB_R19C1.S0.F2  net (fanout=1)          3.908R    N_A<3>
CLB_R19C1.S0.X  Topx                     0.727R    BLK2_2
                                         C56
                                         C55
CLB_R24C1.S1.F4  net (fanout=1)          1.304R    syn1571
CLB_R24C1.S1.X  Topx                     0.727R    BLK1_2
                                         C54
                                         C53
CLB_R20C1.S0.F1  net (fanout=1)          1.562R    syn1574
CLB_R20C1.S0.X  Topx                     0.727R    BLK0_2
                                         C52
                                         C51
P47.O        net (fanout=1)          1.487R    N_Y
P47.PAD      Tioop                    4.787R    Y
                                         C_Y
                                         Y.PAD
-----
Total (7.736ns logic, 8.261ns route)      15.997ns
      (48.4% logic, 51.6% route)
-----

```

5.2. SubBytes Reports

5.2.1. Area

Design Summary

```

-----
Number of errors:      0
Number of warnings:   32
Number of Slices:     71 out of 3,072    2%
Number of Slices containing
  unrelated logic:    0 out of 71    0%
Number of Slice Flip Flops:    8 out of 6,144    1%
Number of 4 input LUTs:    137 out of 6,144    2%
Number of bonded IOBs:    18 out of 166    10%
Number of GCLKs:         1 out of 4    25%
Number of GCLKIOBs:      1 out of 4    25%
Total equivalent gate count for design: 979
Additional JTAG gate count for IOBs: 912

```

5.2.2. Timing

Xilinx TRACE, Version D.19
Copyright (c) 1995-2000 Xilinx, Inc. All rights reserved.

```
trce subbytes.ncd subbytes.pcf -e 3 -o subbytes.twr
```

```
Design file:          subbytes.ncd
Physical constraint file: subbytes.pcf
Device,speed:        xcv300,-6 (PRELIMINARY 1.102 2000-05-03)
Report level:        error report
```

WARNING:Timing:2491 - No timing constraints found, doing default enumeration.

=====
Timing constraint: Default period analysis
65409 items analyzed, 0 timing errors detected.
Minimum period is 27.305ns.

=====
Timing constraint: Default net enumeration
149 items analyzed, 0 timing errors detected.
Maximum net delay is 5.144ns.

All constraints were met.

Data Sheet report:

All values displayed in nanoseconds (ns)

Setup/Hold to clock CLK

Source Pad	Setup to clk (edge)	Hold to clk (edge)
INV	25.531(R)	0.000(R)
RESET	4.748(R)	0.000(R)
XIN<0>	22.402(R)	0.000(R)
XIN<1>	25.379(R)	0.000(R)
XIN<2>	25.358(R)	0.000(R)
XIN<3>	22.300(R)	0.000(R)
XIN<4>	24.932(R)	0.000(R)
XIN<5>	24.525(R)	0.000(R)
XIN<6>	24.727(R)	0.000(R)
XIN<7>	23.715(R)	0.000(R)

Clock CLK to Pad

Destination Pad	clk (edge) to PAD
YOUT<0>	14.607(R)
YOUT<1>	13.583(R)
YOUT<2>	13.969(R)
YOUT<3>	13.679(R)
YOUT<4>	13.580(R)
YOUT<5>	14.466(R)
YOUT<6>	14.420(R)
YOUT<7>	14.053(R)

Pad to Pad

Source Pad	Destination Pad	Delay
INV	YOUT<0>	13.226
INV	YOUT<1>	11.941
INV	YOUT<2>	12.370
INV	YOUT<3>	12.676
INV	YOUT<4>	13.359
INV	YOUT<5>	13.702
INV	YOUT<6>	13.310
INV	YOUT<7>	13.520

Timing summary:

Timing errors: 0 Score: 0

Constraints cover 65409 paths, 149 nets, and 505 connections (100.0% coverage)

Design statistics:

Minimum period: 27.305ns (Maximum frequency: 36.623MHz)

Maximum net delay: 5.144ns

Analysis completed Sun Feb 01 16:55:32 2004

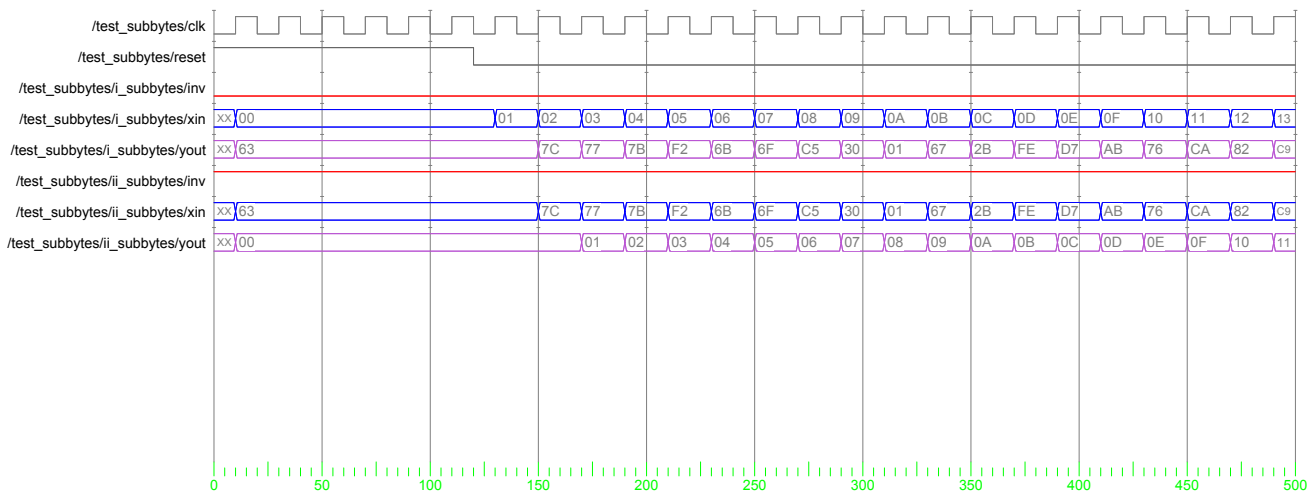
5.3. Synthesis Report Summaries

From parity.vhd timing report (Sec. 5.1), the total critical path delay is 15.997ns within 5 circuit stages. Hence, unit delay = $15.997\text{ns}/5 = \mathbf{3.199\text{ns}}$. Therefore, our design speed equals to $27.305/3.199 = \mathbf{8.53}$ unit delay. The area of the SubBytes reported is equivalent to 979 gates.

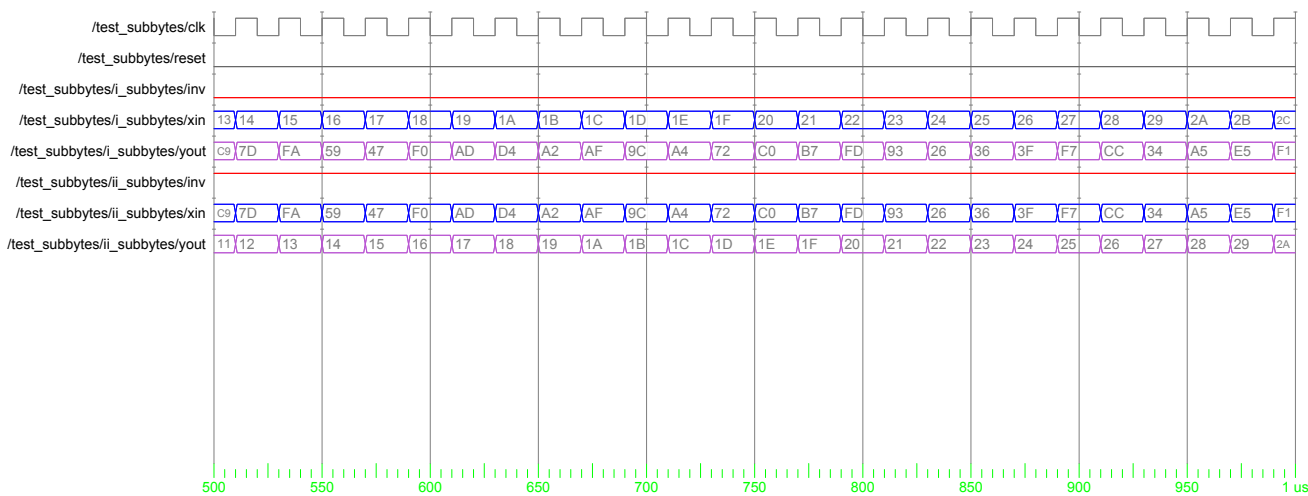
6. References

- [1] A. Rudra et al, "Efficient Galois Field Arithmetic on SIMD Architectures", <http://www.cs.utexas.edu/users/atri/papers>.
- [2] Cillian O'Driscoll, "A Note on Composite Galois Fields and Their Application to Rijndael", December 2001.
- [3] National Institute of Standards and Technology (NIST), "Advanced Encryption Standard," *Federal Information Processing Standards Publication 197*, November 2001.
- [4] Sumio Morioka and Akashi Satoh, "An Optimized S-Box Circuit Architecture for Low Power AES Design".
- [5] Sumio Morioka and Akashi Satoh, "A 10 Gbps Full-AES Crypto Design with a Twisted-BDD S-Box Architecture," *Proc. IEEE International Conference on Computer Design: VLSI in Computers and Processors (ICCD'02)*1063-6404/02, 2002.

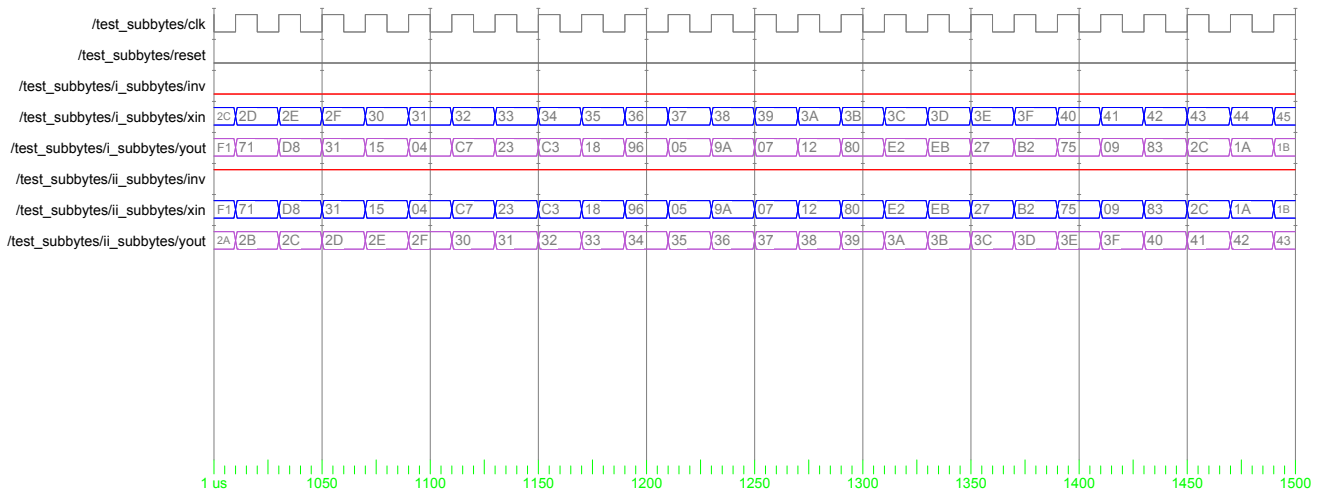
Appendix



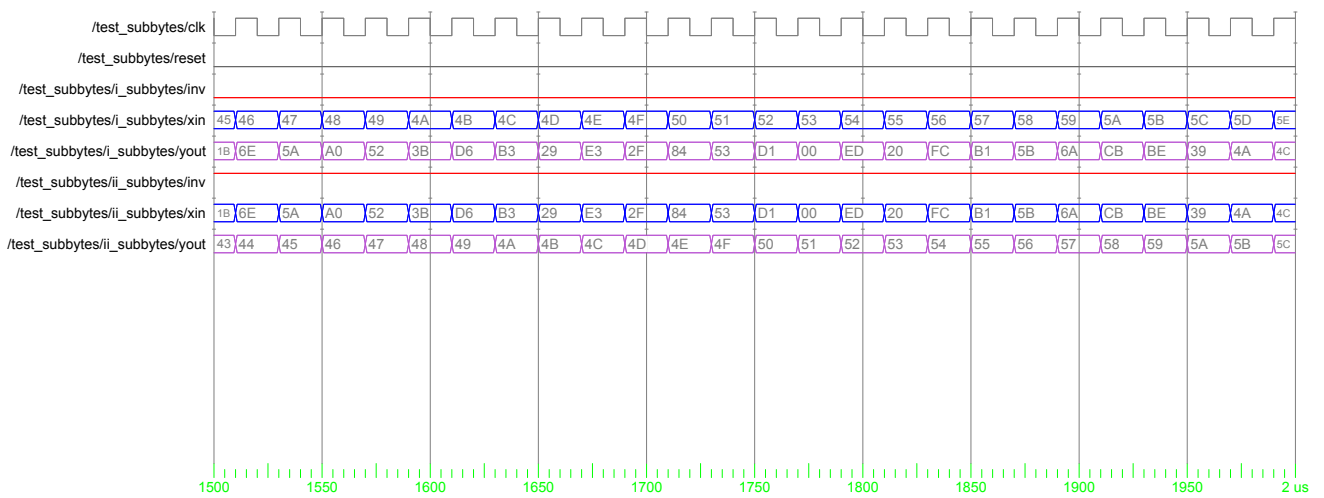
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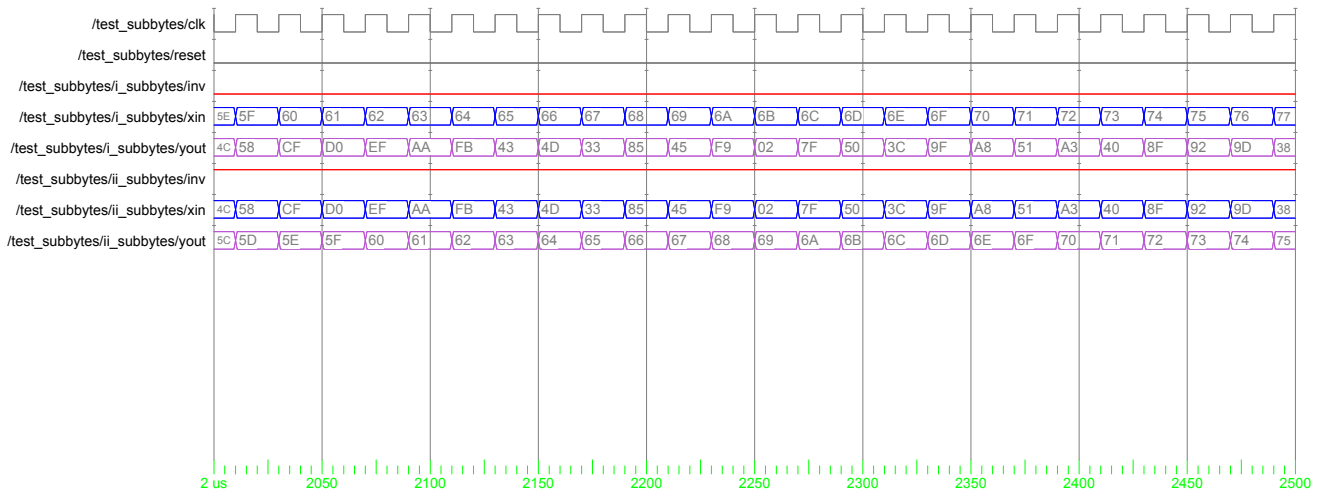
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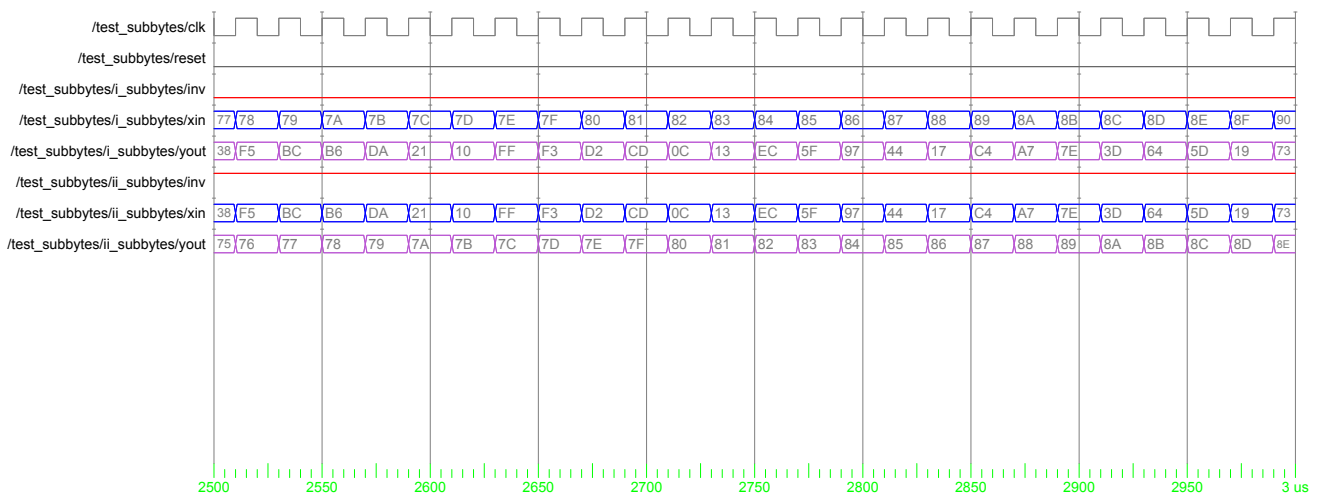
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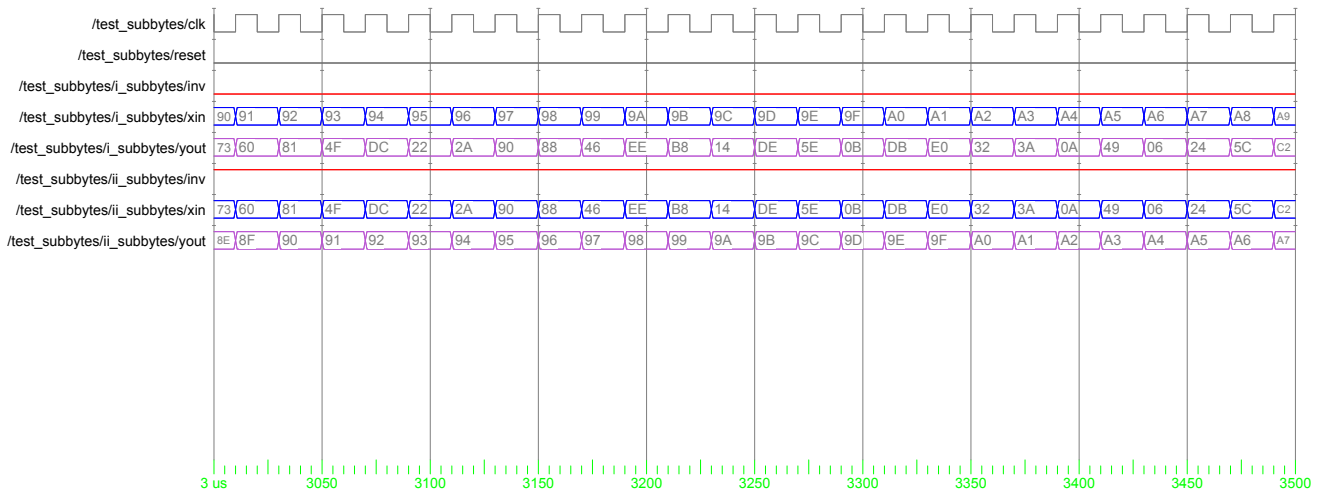
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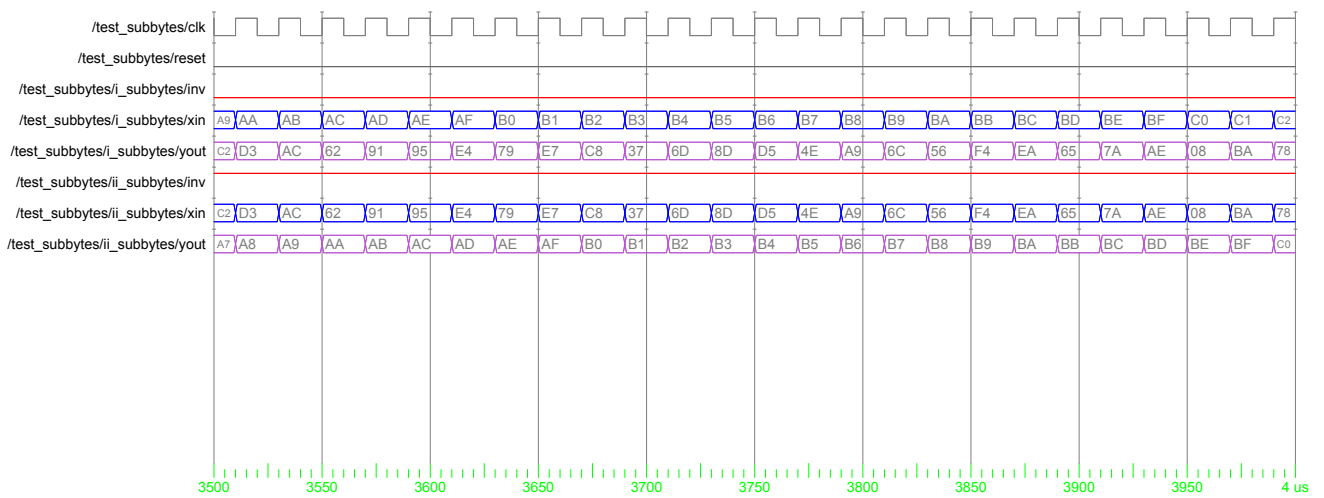
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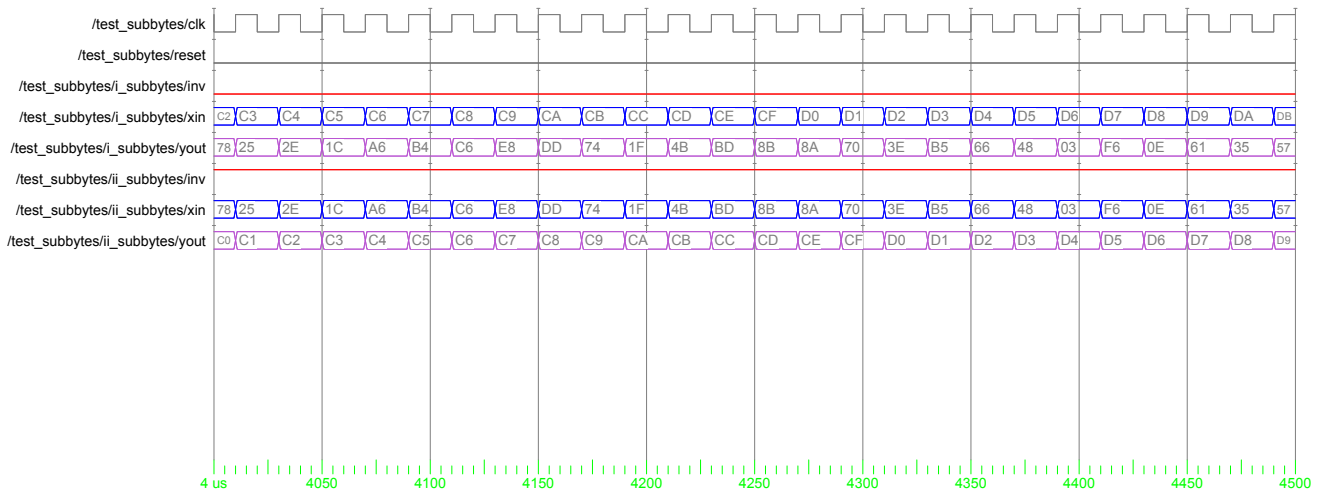
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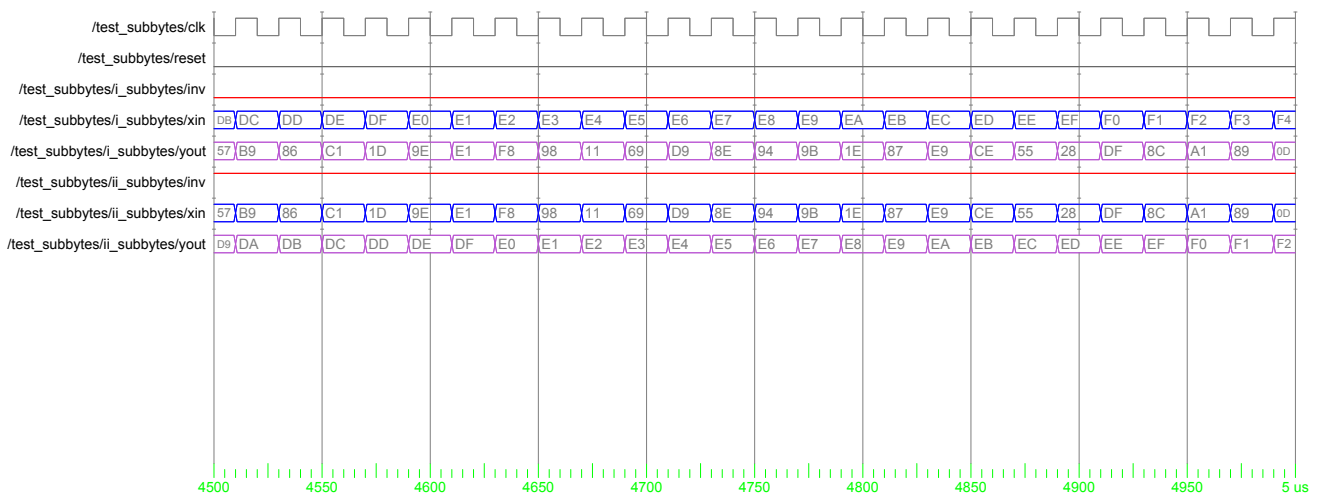
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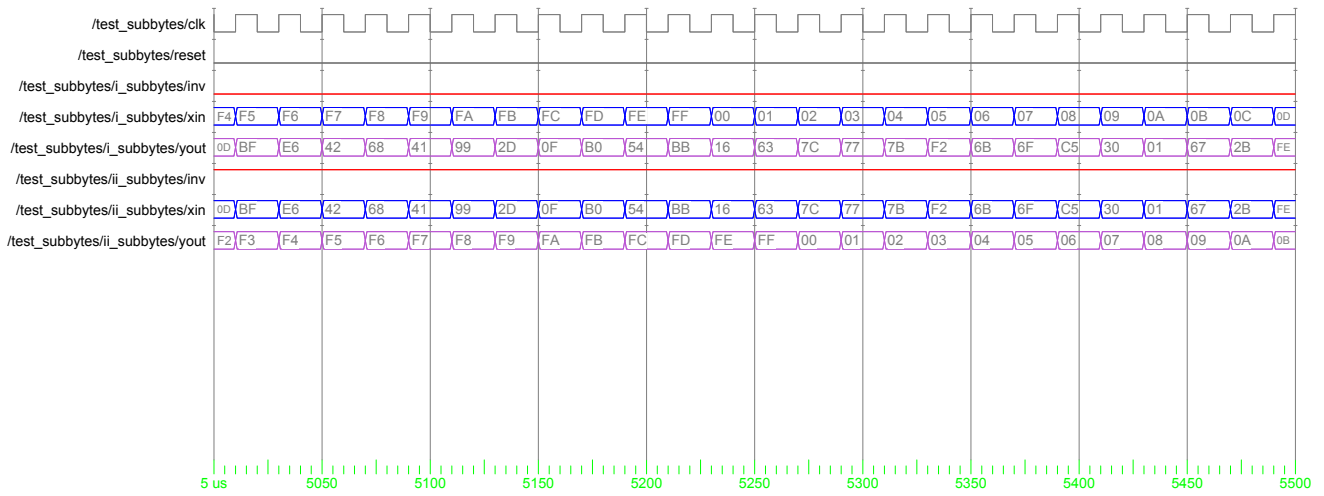
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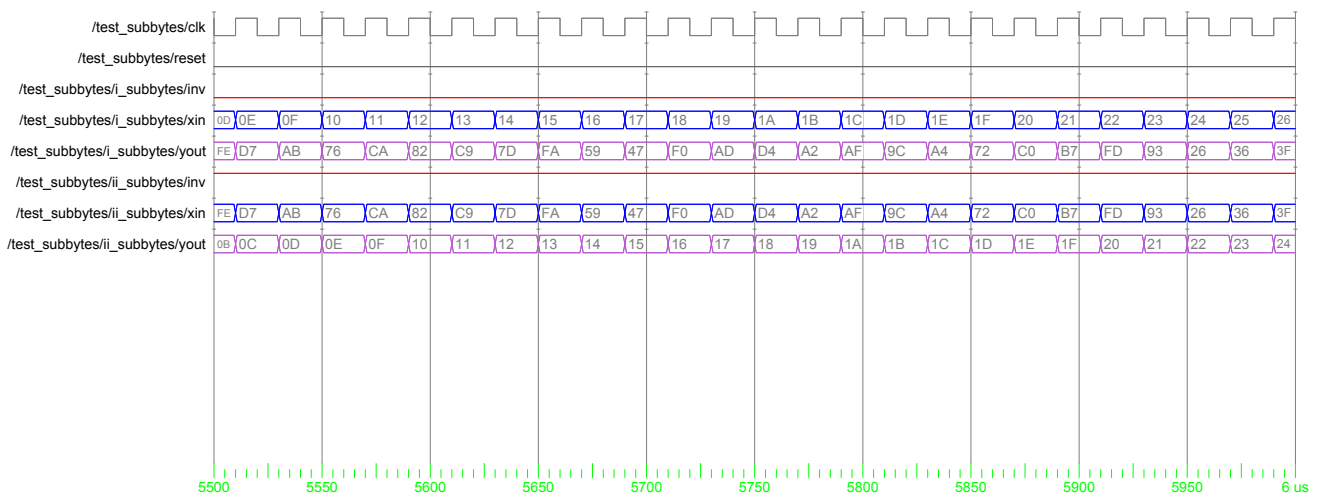
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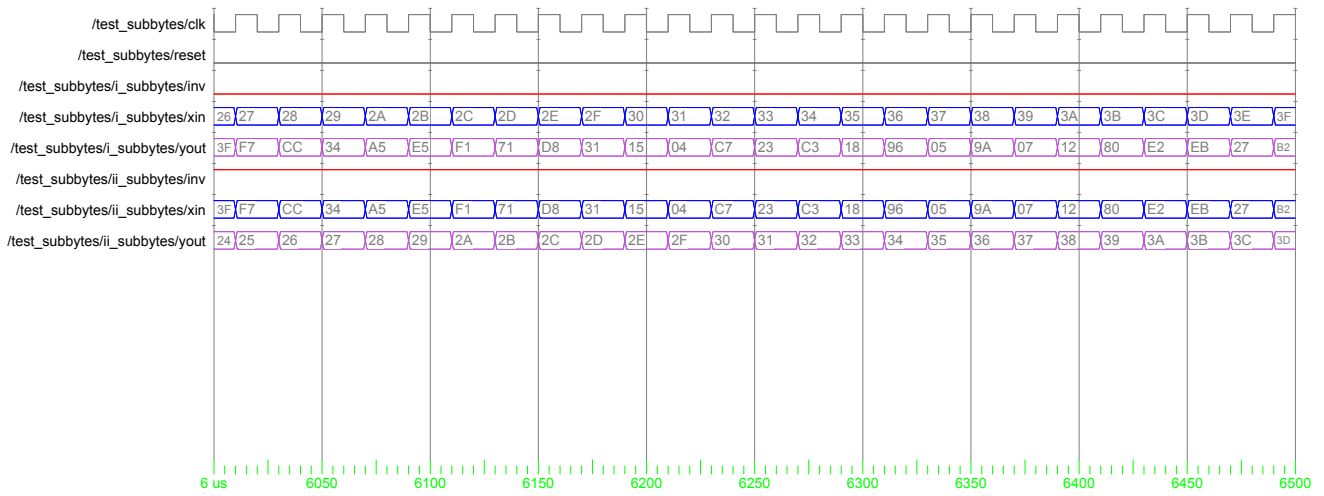
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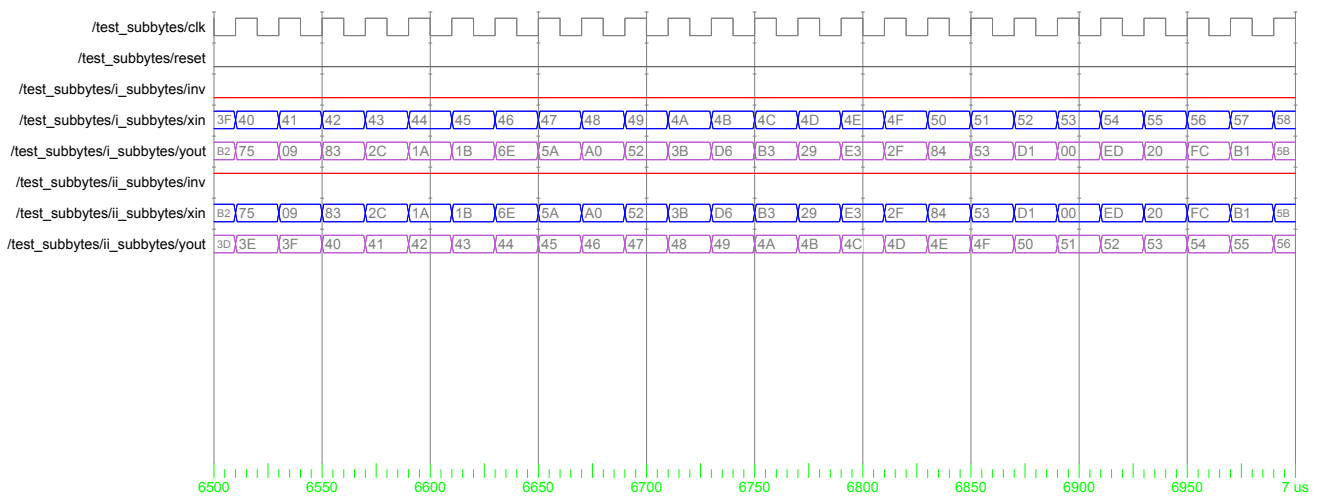
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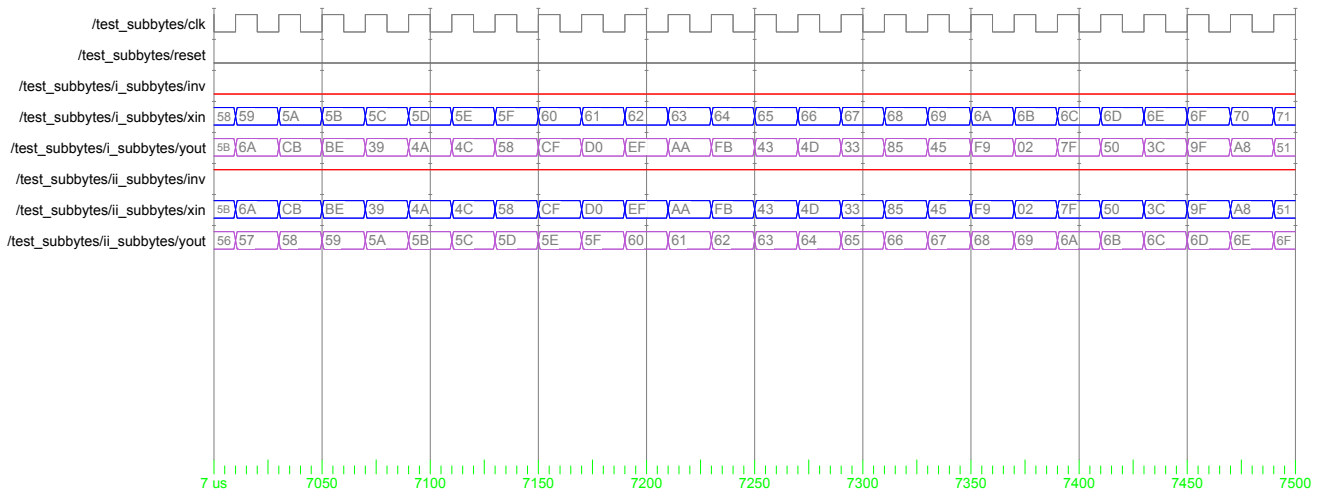
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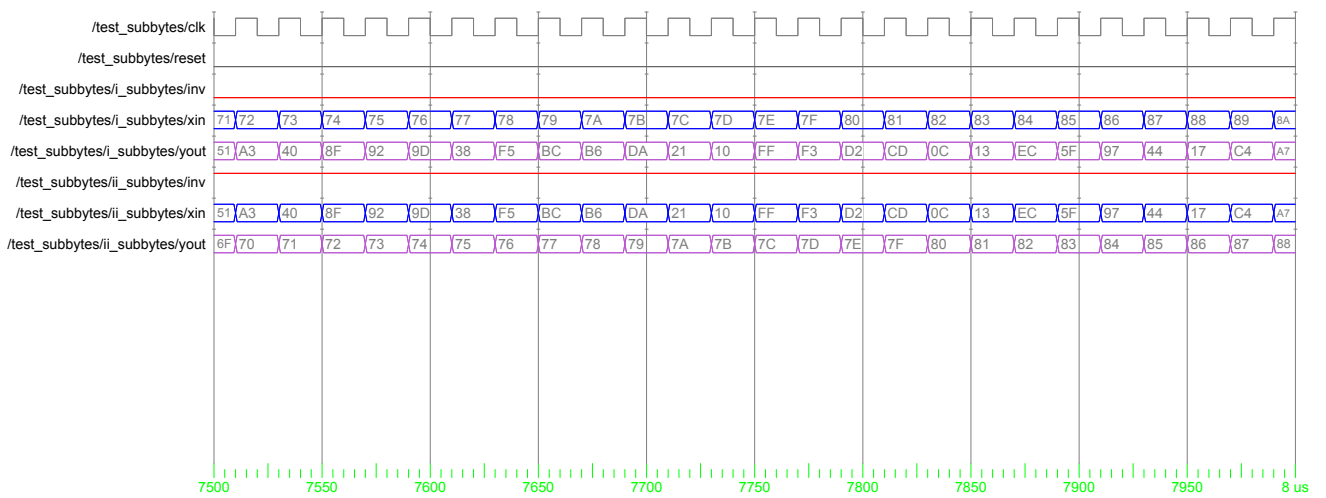
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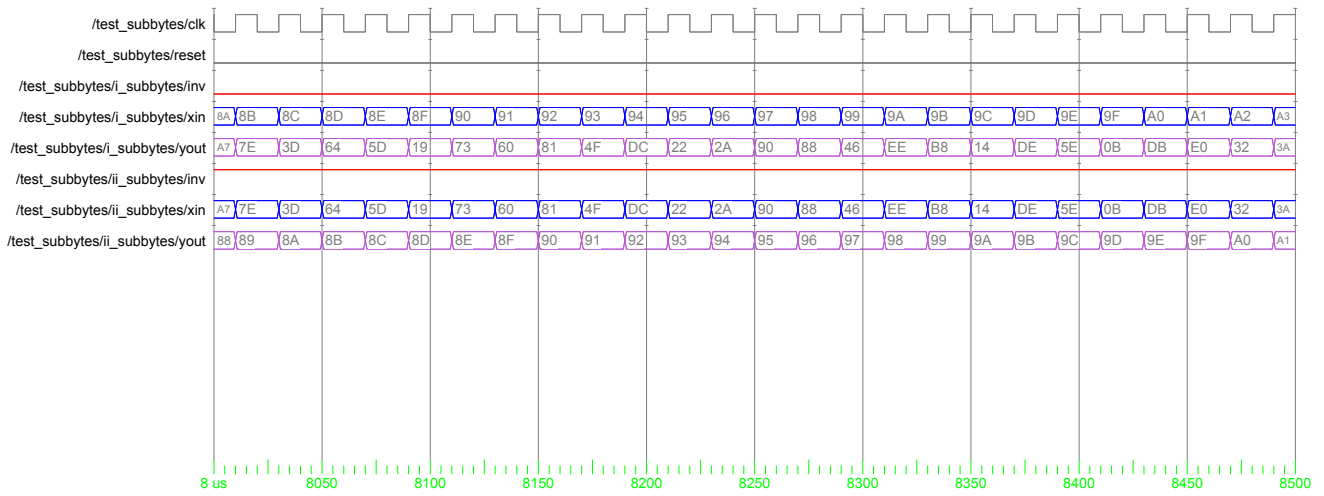
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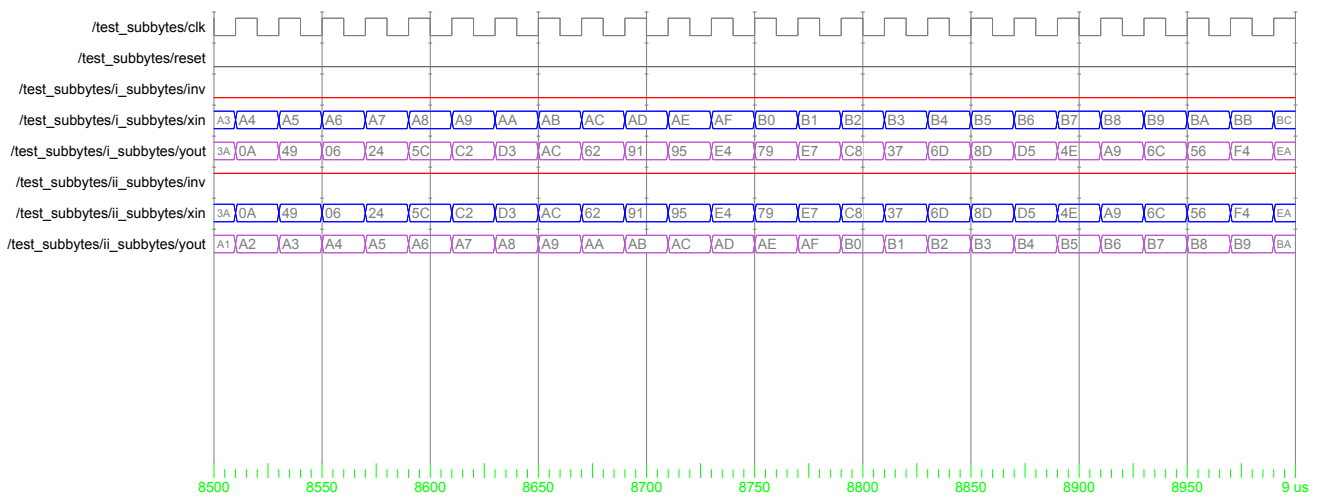
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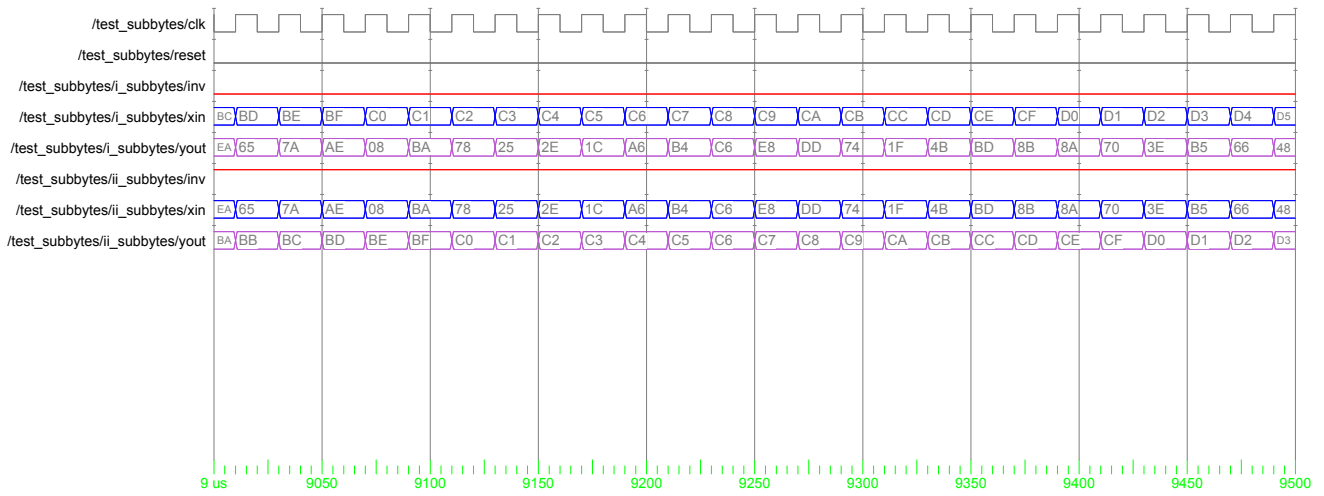
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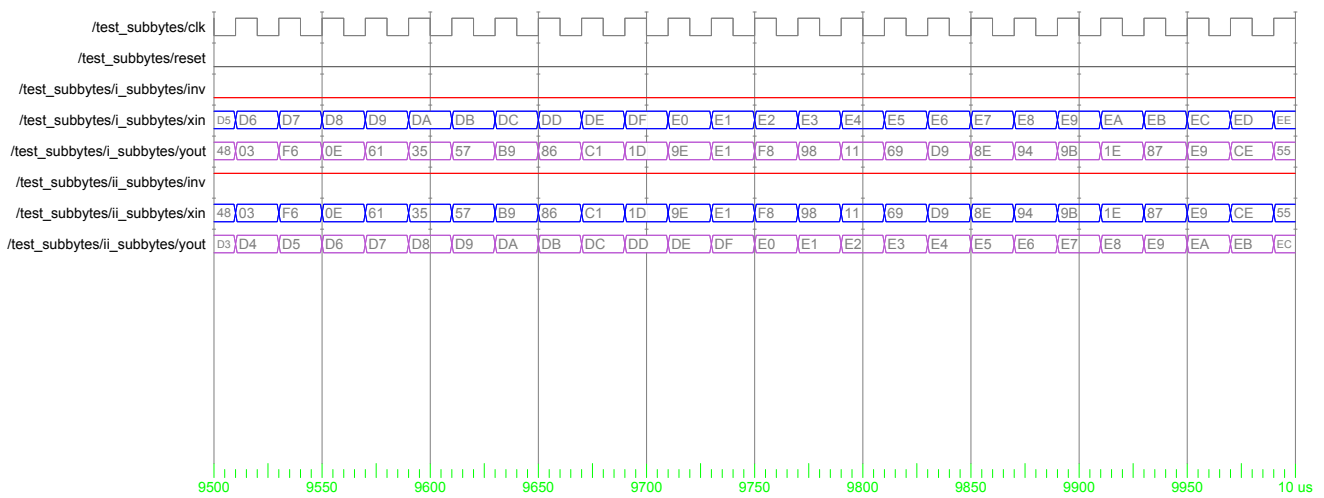
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Entity: test_subbytes Architecture: testbench Date: Thu Feb 05 16:45:20 SE Asia Standard Time 2004 Row: 20 Page: 20